Deformation characteristics and critical state of sand
砂土的变形特性与临界状态

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Abstract: A series of triaxial compression tests were performed to investigate the deformation characteristics and critical state of sand. The test results show that subjected to shear, a sand will exhibit either dilative or contractive behavior. Whether a sand dilates or contracts depends on the current state of the sand, that characterized by both the density of the sand and the effective mean normal stress applied. The larger the density, the lower the effective mean normal stress, the more dilative response is. It is also found that at large strain levels, critical state will appear in all the soil samples tested, and the critical state line is unique irrespective of drainage condition.

Key words: critical state; dilative behavior; effective mean normal stress; drainage condition; sand
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摘 要: 通过一系列室内三轴压缩试验研究了砂土的变形特性与临界状态。试验结果显示砂土在剪切过程中会产生剪胀或剪缩,其胀缩性由该砂土的当前状态所决定,而砂土的当前状态取决于本身的密度和所施加的有效平均正应力。土的密度越大,有效平均正应力越低,土的剪胀性就越大。试验还发现当剪应变水平较高时土样都出现了临界状态,而所观测到的临界状态线与排水条件无关。

关键词: 临界状态; 剪胀性; 有效平均正应力; 排水条件; 砂土

0 Introduction*

In general, subjected to shear dense sand dilates and loose sand contracts. Whereas a large quantity of test results shows that the response of a sand depends not only on its density but also on the effective mean normal stress applied. In particular, effective mean normal stress modifies the behavior of sand to the extent that even a very dense sand, tested at sufficient high pressure, will behave similarly to a loose sand, and vice versa.

Roscoe and Poorooshasb[1] were among the first to emphasize the importance of the combined influence of density and mean normal stress on soil behavior. Later, Been and Jefferies, Bolton, Ishihara, Verdugo and Ishihara, and Cai[2~6] have reported their findings on the combined effects of the density and effective mean normal stress on sand response. Even though, researchers become more and more interesting on this subject.

The critical state is a very important concept in soil mechanics, and a lot of soil models are all based on it. For clay, a well defined straight line in $e-\ln p'$ plane characterizes the final stage of soils, whereas it's not the case for sand, and whether the critical state line is unique for sand is still arguable so far.

In this study, a series of triaxial compression tests were carried out to investigate the deformational behavior and critical state of Leighton Buzzard sand. The tests were performed under various loading and drainage conditions with a relatively large range of densities and effective mean normal stresses. The overall behavior of the sand under the triaxial compression loading condition was evaluated, and the critical state lines in $e-\ln p'$ plane as well as in $q-p'$ plane were obtained.

1 Test apparatus

The CKC triaxial system was adopted for most of the tests, which is composed of a loading frame (with a loading piston mounted on the top), a triaxial cell, a controllable pneumatic loading unit, a signal conditioner, a process interface unit, and a computer with the software installed for system control and data collection. The setup of the system is shown in Fig. 1.

The most remarkable feature of the apparatus is its loading and pressure application system. The pneumatic loading unit is used for regulating the cell pressure and the back pressure as well as the pressures in the piston. By adjusting the pressure difference between the upper and lower chambers in the piston, either compression or tension loading can be applied to the soil specimen.

Fig. 1 CKC triaxial apparatus

The whole process of a test is controlled and monitored by a computer program. Using the modern digital control technique, the following functions can be performed automatically:

Back pressure saturation / B- value check
Isotropic consolidation
Monotonic loading test
Cyclic loading test
Dynamic loading test
K_0 - line consolidation
Constant p test

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Creep and relaxation

Arbitrary stress path test

All tests can be performed either with strain control or with stress control, depending on the usage.

2 Sample preparation

The sand used for all tests is the Leighton Buzzard sand, which is classified as a fine and uniform uncrushed silica sand whose grain sizes are between 0.09 and 0.15 mm. The uniformity coefficient is around 1.3, and the specific gravity of the particles is 2.65. The maximum and minimum dry densities are 1.59 and 1.32 g/cm^3, respectively.

The three most common methods used for sample reconstitution are water sedimentation, dry deposition and moist tamping. The water sedimentation and dry deposition methods can prepare very uniform specimens, but the void ratios of the specimens are not easy to control, and very loose samples cannot be attained. On the contrary, the moist tamping method can prepare very loose samples due to the capillary effects between grains. Of course, it can also be used to prepare medium dense to dense specimens. Besides, the void ratio is very easily controlled. The only drawback is that the specimen is less uniform than that prepared by the water sedimentation or dry deposition method. In this study, the range of void ratio is concerned, so all specimens are reconstituted by the moist tamping method. The sample preparation procedure is described below.

Oven dried sand is first weighed and mixed with de-aired water at a water content of around 5%, and then sealed in a plastic sack for one hour for the equilibrium of moisture. After that, the soil is divided into ten equally-weighted portions ready for tamping.

A membrane with a thickness of about 0.3 mm is stretched taut to the inside face of a split mold which is fixed to the base of a triaxial cell by an "o" ring. Each portion of the moist sand is strewed with a tiny spoon into the mold, and then a small hammer is used to tamp the soil gently to a predetermined height. It should be noted that the tamping energy may affect the fabric of soils, so in the procedure, the same energy (same falling height) is applied. In such a way the amount of tamping for a dense specimen is more than that for a loose sample. After the tamping is finished, the surface of the soil is scraped, and the next layer of soil is strewed in. When all soil layers are tamped, the specimen is enclosed by the membrane with the top cap on. The top drainage line is then connected. At this time, a very small vacuum (5 kPa) is applied from the top drainage line to hold the specimen, and the mold is removed.

Carbon Dioxide is percolated through the sample from its bottom to the top. Normally this process takes about half an hour. After most of the air in the specimen is replaced by Carbon Dioxide, a vacuum of 15 kPa is applied through the upper and lower drainage lines of the setup.

De-aired water is circulated though the specimen from the bottom drainage line to the top. This process should be performed very slowly, otherwise the air bubbles could not be carried out through the water. The saturating procedure of the specimen is the key step for a test, as the degree of saturation not only influences the effective stress measured, but also affects the void ratio evaluated. The water circulation process normally lasts for 4–6 hours.

After a specimen is prepared and the cell had been set up on the loading frame, an initial B-value check is made. For most of the tests the B-values are over 0.8 at this stage. To raise the degree of saturation, a back pressure is applied to the sample through the drainage line, and during this process the effective stress is kept constant by simultaneously increasing the cell pressure by the same amount. The B-value is checked at each step of the increment. Normally a back pressure of 100 to 200 kPa is enough, with which the B-value can easily reach 0.98 or even higher.

3 Evaluation of initial void ratio

Two methods are available for evaluating the initial void ratio. The first is a direct method, in which the void ratio is evaluated by measuring the dimension of the specimen directly at the time when the water circulation process is finished, then the measured value plus the void ratio increment recorded in the consolidation process is taken as the initial void ratio for a shear test. The second is an indirect method, in which the void ratio is obtained from the back calculation based on the final measurement of water content of the specimen. After shearing is finished, the specimen is frozen, and then the water content can be measured. By assuming that the soil is fully saturated, the final void ratio can be calculated by $e = G_s \cdot w$. In undrained tests, this value is taken as the initial void ratio, as no volume change is allowed during the shear process; in drained tests, the initial void ratio is the sum of this value and the void ratio increment during shearing. In this study, the direct sample dimension measurement method was adopted for all the tests. The water content measurement method was also used in a few tests for the purpose of comparison. It is found that the deviation of the void ratios measured from these two methods is very small. The study also shows that a low initial degree of saturation (before back pressure saturation) may be a key factor causing the discrepancy. When the specimen is not well saturated during the water circulation, a higher back pressure should be used to saturate the soil specimen. During this process, a large amount of water can flow into the specimen. This could induce a volume change of the specimen a lot.

4 End caps

It is well known that the response of the soil in triaxial tests at high strain level is significantly influenced by the friction between the end cap and the soil specimen. To promote a relatively homogeneous deformation, the so-called "free-end" with enlarged platens was adopted for all the tests. The lubricating system on the end platens consists of one sheet of a 0.23 mm thick latex membrane coated with a layer of silicone grease on the side facing the aluminum plate. Besides, the diameter of the end platens is larger than the diameter of the specimen such that the end of the soil specimen is always within the end platen during testing. The test results show that the "free-end" system can produce more uniform deformation field than the conventional end platens do.

5 Deformation characteristics of sand under triaxial compression loading condition

Table 1 shows the test conditions for all the tests. Here u_B is the back pressure applied, and those B-values are all measured after consolidation. A total of 18 tests were performed, including 9 undrained tests and 9 drained tests.

Table 1 Conditions of Triaxial Compression Tests

No.	Kind of test	p_0' /kPa	e_0	D_r /%	u_B /kPa	B-Value
1			0.992	4.68	500	0.876
2		100	0.912	28.2	500	0.914
3			0.886	35.8	800	0.994
4			0.983	7.33	550	0.992
5	CU	500	0.894	33.5	500	0.986
6			0.864	42.2	500	0.991
7			0.97	11.1	500	0.981
8		1000	0.883	36.7	500	0.990
9			0.848	46.9	500	0.993
10			0.999	2.64	500	0.992
11		100	0.926	24.0	500	0.985
12			0.869	41.8	500	0.992
13			0.962	13.5	500	0.995
14	CD	500	0.915	27.3	500	0.983
15			0.897	37.8	500	0.994
16			0.928	23.5	500	0.987
17		1000	0.901	31.4	500	0.997
18			0.85	46.3	500	0.987

For each test, the soil specimen was first consolidated to a predetermined initial state (including both void ratio and effective mean normal stress), and then a compressive deviator stress was applied. All the tests were conducted under the strain- controlled condition of loading with a deformation rate of around 0.05 mm/min. The cell pressure was kept constant during shearing, so the total stress path was along a line of $dq/dp' = 3$, where $p' = (\sigma_1' + 2\sigma_3')/3$ is the effective mean normal stress, and $q = \sigma_1' - \sigma_3'$ is the deviatoric stress. The axial load, pore water pressure, volume change (in drained tests) and axial deformation were measured every 25 seconds.

In data interpretation, two stress invariants q and p' and two strain invariants ε_q and ε_v are referred. The deviatoric strain and volumetric strain are defined as $\varepsilon_q = 2(\varepsilon_1 - \varepsilon_3)/3$ and $\varepsilon_v = \varepsilon_1 + 2\varepsilon_3$, respectively.

Figs. 2a and 2b show the undrained test results of three soil specimens with different densities subjected to the same initial effective mean normal stress $p_0' = 100$ kPa. It can be seen that under the compression loading, the sand exhibits two kinds of response: the contractive response and dilative response. When the soil is very loose, i.e., $e_0 = 0.992$, the contractive response is seen. The mean normal stress continuously decreases, while the deviatoric stress first increases to a peak, and then quickly decreases until reaching a stable value at large strains, where the soil sample may deform continuously under a constant shear stress and constant mean normal stress. This state is the well- known critical state, which will be discussed in detail later. During the whole process positive pore water pressure is observed. On the contrary, when the soil is medium dense to dense (i.e., $e_0 = 0.912$ and 0.886), the dilative response appears: the mean normal stress first decreases moderately, and approaches a "phase transformation" state in which the rate of the mean normal stress change is zero. After this state the effective mean normal stress increases. In accordance with the variation in mean normal stress, the pore water pressure increases and decreases later, eventually it may enter a negative range. The deviatoric stress always increases up to an ultimate value achieved at a large strain. It should be noted that the effective stress path along the dilative shear part is curved inwards, so the maximum stress ratio (q/p') is not necessarily associated with the ultimate state.

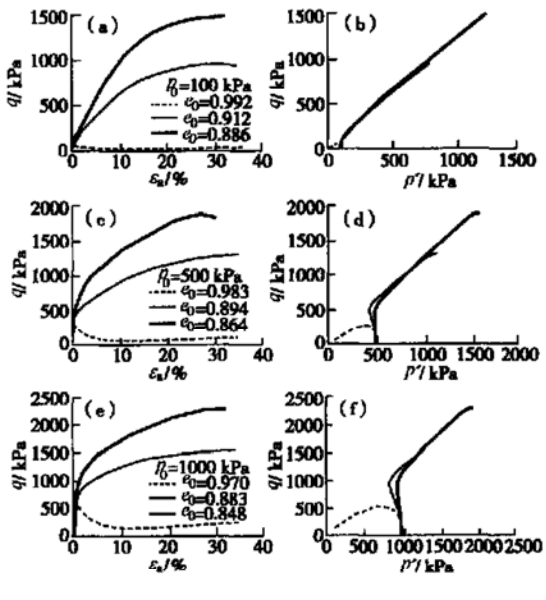

Fig. 2 Undrained trizxial compression test

Figs. 2c to 2f show the other undrained test results with initial mean normal stress $p_0' = 500$ and 1000 kPa, respectively. Similar behavior of sand has been observed. It reveals that, subjected to undrained shear, the sand will show either contractive or dilative behavior; under the same initial effective mean normal stress, the higher the void ratio of the specimen, the more contractive the response is. On the other hand, for the same void ratio, the higher the confining pressure is, the more the contractive response appears. The tests proved the classical observation that the contractive or dilative response of sand depends not only on the void ratio of the soils but also on the effective mean normal stress applied.

Figs. 3a and 3b shows the drained compression test results with an initial mean normal stress $p_0' = 100$ kPa. For the loose sample ($e_0 = 0.999$), the sand exhibits the strain hardening behavior: the deviatoric stress continuously increases with the deviatoric strain until reaching an ultimate state, correspondingly the volume of the soil specimen monotonically reduces. For the dense sample ($e_0 = 0.869$), the strain softening behavior is observed: the deviatoric stress first increases to a peak, and then decreases towards an ultimate state. The volumetric strain is positive at low strain levels and then becomes negative, indicating a volume expansion. It should be noted that, under the same initial effective mean normal stress, despite the large difference in the stress- strain behavior, the samples tend to have the same ultimate strength. In addition, the volume of the sample becomes nearly constant finally. This ultimate state agrees with the definition of the critical state in critical state soil mechanics.

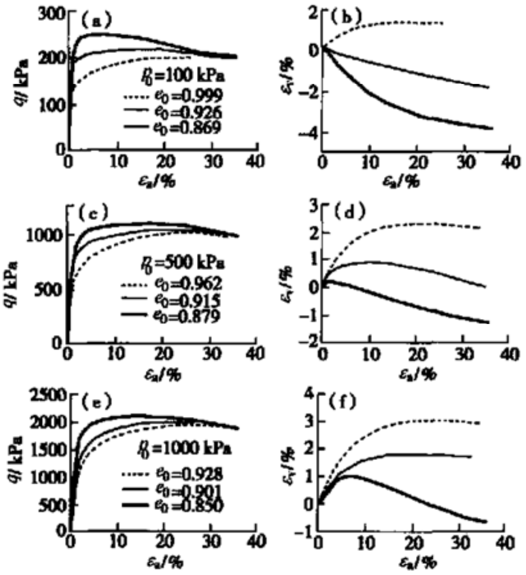

Fig. 3 Drained traxial compression test

Figs. 3c to 3f show the other drained test results with initial effective mean normal stress $p_0' = 500$ and 1000 kPa, respectively. Similar behavior of sand has been observed. They further indicate that the void ratio and effective mean normal stress join together to determine the response of sand.

6 Critical state of sand

It is observed that, irrespective of the initial state, sand sheared under either drained or undrained conditions will finally reach a unique ultimate state, where effective mean normal stress and deviatoric stress are constant, and the increment of volumetric strain is zero, only the deviatoric strain continues to develop. This ultimate state is known as the critical state (Roscoe et al, 1958[7]) or steady state (Poulos, 1981[8]).

As pointed out before, under both drained and undrained conditions, the critical state has been reached for most of the tests presented when the deviatoric strain exceeds 25%. It can be seen that in some of the tests the effective mean normal stress and deviatoric stress as well as the volume (in drained condition) still change after the strain exceeds 25%. However, the increments of these quantities are small and the rates of change are decreasing. So in those cases, the state (both void ratio and stress) of specimen at the deviatoric strain = 25% are treated as the critical state.

Fig. 4a shows the relations between the measured effective mean normal stresses p' , and the void ratios, e , at the initial state as well as at the ultimate critical state defined by the deviatoric strain = 25%. Both the undrained and drained tests are included. The solid points denote the critical states and the hollow points denote the initial states. A trend line of the critical states in the $e - \ln p'$ plane is drawn. It can be seen that the critical state in $e - \ln p'$ plane can be obtained based on a known e or a known p' at the critical state, independent of the initial state of the specimen, and irrespective of the drainage condition. It can also be seen that the critical state line for the sand in $e - \ln p'$ plane is in general a curve, this is quite different from the critical state line observed for clay.

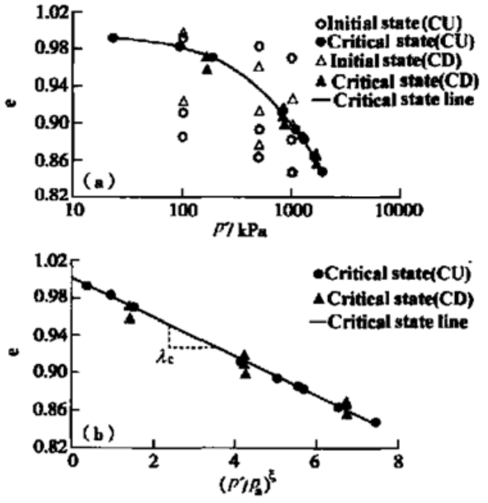

Fig. 4 Critical state line in $e - p$ and $e - (p'/p_a)$ plane

To obtain a straight critical state line, Li and Wang[9] used $(p'/p_a)^\xi$ to normalize p' , and found a straight line in $e - (p'/p_a)^\xi$ plane for critical state, which can be expressed

$$e = e_\Gamma - \lambda_c \cdot \left| \frac{p_c'}{p_a} \right|^\xi \tag{1}$$

where, p_a is the atmospheric pressure for normalization. ξ is a positive constant, and e_Γ and λ_c are the critical void ratio intercepting the $p' = 0$ axis and the slope of the critical state line, respectively. For the Leighton Buzzard sand tested here, $\xi = 0.68$, $e_\Gamma = 0.998$, and $\lambda_c = 0.0198$. Eq. 1 represents an important reference for evaluating sand response.

Fig. 5 shows the effective mean normal stress versus the deviatoric stress, measured at the critical state. It can be seen that in $p' - q$ plane the critical stress ratio is basically a straight line passing through the origin. The slope of the line, M, is found to be 1.185 for the tested sand. It indicates that at critical state a unique friction angle, ϕ_{cri} is mobilized, which is related to M by the following equation:

$$\phi_{cri} = \sin^{-1}\left(\frac{3M}{6+M}\right) \quad (2)$$

With $M = 1.185$, the mobilized friction angle at critical state in triaxial compression is found to be 29.7°.

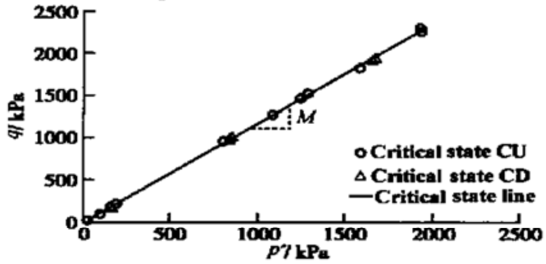

Fig. 5 Critical state line in $p'-q$ **plane**

7 Summary and discussion

Triaxial tests were carried out on saturated Leighton-Buzzard sand to evaluate large deformational behavior of cohesionless soil. The following conclusions are drawn from the present experimental investigation:

(1) It has been shown that dilation is a feature of the sand tested. Subjected to shear, the sand will either dilate or contract. Whether a sand will dilate or contract depends on the initial state of the soil that includes both void ratio and effective mean normal stress. In the undrained shear case, the dilation of sand is reflected in the pore pressure change, i. e., a negative pore pressure will be induced for a sand in dense state and a positive pore pressure is expected for a sand in loose state. In the drained shear case, the dilation of sand is reflected in the volume change, i. e., a negative volumetric strain (dilation) will develop for a dense sand, while a positive volumetric strain (contraction) will develop for a loose sand.

(2) Subjected to monotonic loading, the stress strain response of sand in the undrained case has two different types: strain hardening behavior for dense sand and flow type for loose sand. In the drained case, two different types of stress strain relations have also been observed: strain- hardening behavior for loose sand and strain softening behavior for dense sand. The strain hardening behavior is characterized by a continuous increase of deviatoric stress. While both flow type (undrained cases) and strain softening type (drained cases) stress- strain relations are characterized by a reduction of deviatoric stress after reaching the peak stress, they are fundamentally different phenomena (the stress ratio continuously increases for flow type, while the strain softening behavior is associated with a stress ratio reduction). The former results from contractive behavior of a sand, and the later is associated with a sand's dilative behavior.

(3) Under an undrained or essentially undrained shear condition, the stress path measured in medium to dense sand always crosses the phase transformation state where the sand response changes from contraction to dilation. Correspondingly, the effective mean normal stress changes from decreasing to increasing.

(4) In the tests, dense or loose sand sheared under either drained or undrained conditions, will eventually reach a critical state, and the critical state line is irrespective of drainage condition.

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