Steady state strength of sand: concepts and experiment
砂土的稳态强度：概念与试验

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Abstract Steady state strength is an important parameter in evaluating the stability of sand deposits against flow liquefaction. Most loose sands exhibit the quasi-steady state (QSS) behavior under undrained triaxial loading, in which no unique ultimate steady state is reached. Stress-controlled triaxial tests were carried out on Unimin sand to study the steady state strength in the QSS behavior. The test results indicate that there are usually four distinct deformation stages in undrained tests on loose sand: initial stage, collapse stage, critical stress stage and post failure stage. The strain rate approximately remains constant in each deformation stage. The steady state, in which pore pressure, deviator stress, effective stress and strain rate remain constant, occurs in the collapse stage. The conventional definition of the steady state of deformation is not completely valid for the QSS behavior, and a modified definition is suggested.
Key words liquefaction, quasi-steady state, sand, steady state, strength, testing, triaxial test
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文 摘 残余强度或稳态强度的确定是砂土液化研究中的重要课题。饱和砂土有3种典型的不排水剪切特征：稳态性状、准稳态性状、和加工硬化性状。在三轴不排水剪切中，大部分松砂表现出准稳态性状。新近的研究表明，"准稳态性状"不是砂土的固有性状，而是三轴试验中的边界条件所导致。本文通过试验研究发现，饱和砂土在三轴不排水剪切中通常表现出4个明显不同的阶段：初始阶段、坍塌阶段、临界状态应力阶段、和后破坏阶段。稳态强度只有在坍塌阶段中才会较好地表现出来。文中还对变形稳态的定义作了修正和补充，并根据修正的变形稳态定义给出了Unimin砂的稳态强度及其它的一些试验性质。
关键词 砂土，液化，稳态，准稳态，试验，抗剪强度，三轴试验
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1 Introduction

Steady state strength is an important parameter in evaluating the stability of sand deposits against flow liquefaction and is often determined by triaxial tests. There are three different stress-strain behaviors of saturated sands in undrained triaxial tests: strain hardening behavior, steady state behavior, and quasi-steady state (QSS) behavior [1]. In the strain hardening behavior, the shear resistance always increases with axial strain, and no flow liquefaction occurs in this case. In the steady state behavior, the shear stress rapidly decreases once a collapse is triggered, and then remains constant with further axial strain. The constant shear stress in the steady state is considered as the undrained residual strength or the steady state strength of the sample. The steady state strength of a sand is thus well defined in the steady state behavior.

Most loose sands exhibit the quasi-steady state behavior under undrained triaxial loading [2~4]. In the quasi-steady state (QSS) behavior, a sample also deforms rapidly during collapse, and the shear stress reaches a minimum value and remains at the minimum value until phase transformation (PT); subsequently, the shear stress increases with further strain. In other words, no unique ultimate shear resistance exists in the QSS behavior. This behavior has led some researchers to propose the terms "limited liquefaction" or "flow with limited deformation" (e.g. literature [3]). By definition, the steady state of deformation is achieved only after all particle orientation has reached a statistically steady state condition and after all particle breakage is complete [5]. The ultimate steady state might not be reached in the QSS behavior within the strain achievable in the triaxial apparatus [2,6]. Verdugo and Ishihara [7] suggested two steady states in the QSS behavior, a temporary steady state occurs during collapse, and the real steady state occurs at large axial strain. Thus, determination of steady state strength in the QSS behavior appears to be difficult. However, Zhang and Garga [8] found that the QSS behavior may not be an inherent behavior of sands, since the post-phase transformation increase in shear stress is primarily caused by end restraint and undrained volume change. Therefore, a further examination of the steady state strength in the QSS behavior appears to be necessary.

2 Experimental investigation

Unimin sand 2010 from St-Canut, Quebec, Canada was used in this study. It is an angular medium crushed quartz sand. The mean grain diameter d_{50} is 0.87 mm and the coefficient of uniformity ($C_u = d_{60}/d_{10}$) is 2.00. The specific gravity G_s is 2.665. The minimum void ratio of 0.646 and the maximum void ratio of 1.027 were determined according to the ASTM test methods D 4253-93

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(Method 1A) and D 4254-91 (Method A), respectively.

A computerized triaxial apparatus was used to conduct conventional undrained and drained triaxial tests on the tested material under stress controlled conditions. A Hall Effect radial displacement transducer (HRDT) was used to measure the radial deformation of samples during saturation and consolidation. The samples were prepared by using the moist tamping method. Their volume changes during saturation and the membrane penetration during consolidation were determined for each sample by measuring the axial and radial deformations. Lubricated end platens were used in all tests. Shearing was conducted under stress-controlled mode with a stress rate of 10 kPa/min. Pore pressure and vertical deformation during rapid collapse were recorded by using a dynamic signal analyzer (DSA).

Twenty-two drained and undrained triaxial tests were carried out on Unimin sand. The results of three typical undrained tests (designated as (1) to (3)) and two typical drained triaxial tests (designated as (4) and (5)) on Unimin sand are shown in Figures 1 to 5, respectively, and will be described in detail subsequently. Their states prior to shear and steady states, as well as the states of the other tests, are presented in Fig. 6. The steady states were determined based on the modified definition of the steady state of deformation proposed in this paper.

3 Quasi-steady state behavior in undrained and drained tests

Typical results of undrained triaxial tests on loose Unimin sand are shown in Fig. 1. It is clear that the sample exhibited the quasi-steady state behavior. When the deviator stress reached a peak value at an axial strain of 1.6%, the sample suddenly collapsed and deformed to 15.6% strain in about one second with a deviator stress drop of 106 kPa (Fig. 1). Subsequently, the deviator stress continuously increased to 591 kPa at an axial strain of 25% and essentially remained constant with further strain. Fig. 1 (b) shows the strain and pore pressure changes during collapse. Before collapse, the sample strained slowly, at a strain rate of about 1.8% per minute. Once the collapse was initiated, its strain rate suddenly increased to about 900 % per minute, and essentially remained constant during collapse, and subsequently decreased to 3.6 % per minute. The pore pressure increased very rapidly, from 752 kPa to 950 kPa when collapse was initiated, and then remained constant during the collapse; subsequently, it continuously decreased until a strain of 25%, and essentially remained constant thereafter. Thus, a temporary steady state was achieved between axial strains of 9% and 15%, and another steady state also occurred after a strain of 25% in the test. The deviator stress in the later steady state was 590 kPa, much larger than that of 456 kPa in the temporary steady state. The stress path in the test was located on a straight line after collapse (Fig. 1 (c)).

Fig. 2 shows the typical results of triaxial drained tests on loose Unimin sand. It can be seen that the deviator stress of the sample increased with axial strain and reached an ultimate value of 138 kPa at a strain of 12.5%, and essentially remained constant thereafter. The void ratio decreased from a value of 0.969 prior to shear to a value of 0.921 at a strain of 11.5%, then remained constant until a strain of 17%; subsequently, the void ratio continuously increased to 0.933 at the end of the test. The strain rate gradually increased with strain until the ultimate deviator stress was reached, and then remained constant. It is clear that by definition, a temporary steady state of deformation was achieved when the void ratio, the deviator stress and the strain rate of the sample stayed constant between strains of 12.5% and 17%. It is also noted that beyond a strain of 17%, the void ratio of the sample continuously increased, even though the deviator stress and the strain rate remained constant. This kind of drained behavior corresponds to the quasi-steady state behavior in undrained tests in which the deviator stress continuously increases with further strain after phase transformation.

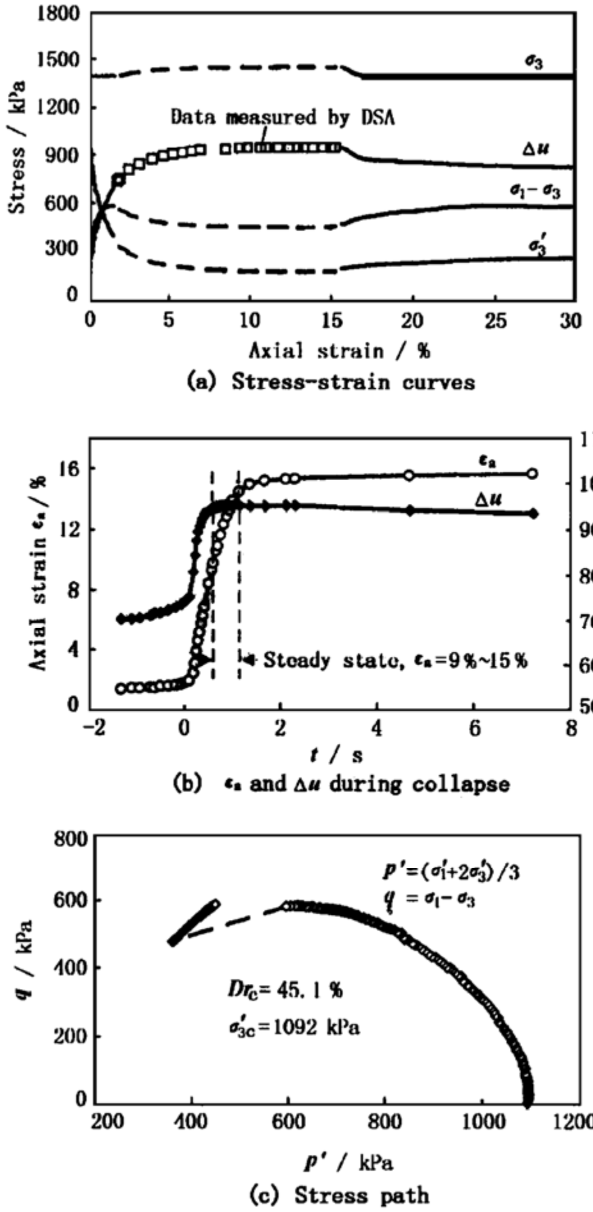

Fig. 1 Typical results of undrained test on loose sand (sample (1))

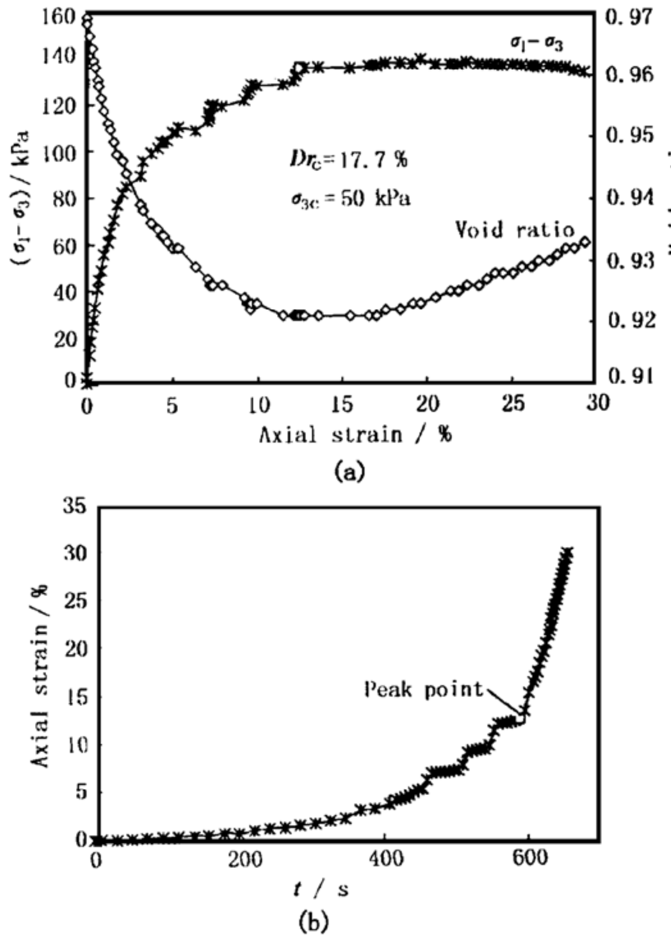

Fig.2 Typical results of drained tests on loose sand (sample 4))

4 Deformation stages during shear

4.1 Undrained tests

Fig.3 shows typical results of an undrained test on a sample which has a consolidation state close to and above the steady state line of the sand. It can be seen in Fig.3 (a) that in the test, the deviator stress increased to a value of 156 kPa at a strain of 0.9% (point a), and remained at this value up to a strain of 4.2% (point b) when the pore pressure reached a peak value of 135 kPa at phase transformation. Subsequently, the deviator stress continuously increased to 322 kPa at a strain of 28% (point c) and remained constant again thereafter; while the pore pressure decreased essentially along a straight line until the end of the test (point d).

Sample 2) exhibited four distinct deformation stages with constant strain rates during undrained triaxial shear, namely initial stage, collapse stage, critical stress stage and post failure stage (Fig.3 (c)). It is seen that in the initial stage, both the deviator stress and the pore pressure increased with strain, and the strain rate was low, equal to about 0.08% per minute. At the end of the initial stage (point a), the sample suddenly collapsed and entered the collapse stage during which it deformed from a strain of 0.9% to a strain of 4% (point b) at a faster and constant strain rate of 7.6% per minute. Correspondingly, the deviator stress remained constant during collapse (from point a to point b), and the pore pressure increased to a peak value and then essentially remained constant before the end of this stage (Fig.3 (a)). In the critical stress stage, the sample deformed to an axial strain of 28% (point c) at a low strain rate of 1.7% per minute, accompanied by continuous increase in deviator stress and continuous decrease in pore pressure. This stage is called critical stress stage since the shear stress in the stage is equal to the fully mobilized shear resistance. The stress path in this stage followed a straight line called critical stress path, which is defined as the locus of the stress states when the friction resistance of a sand mass is fully mobilized under given effective stresses. In the post failure stage which started at a strain of 28%, both the deviator stress and the strain rate approximately remained constant, but the pore pressure continuously decreased so that the effective stress state moved away from the critical stress path (Fig.3 (b)). The temporary steady state of this sample seemed to appear shortly before the end of the collapse stage, when the deviator stress, pore pressure and strain rate were constant.

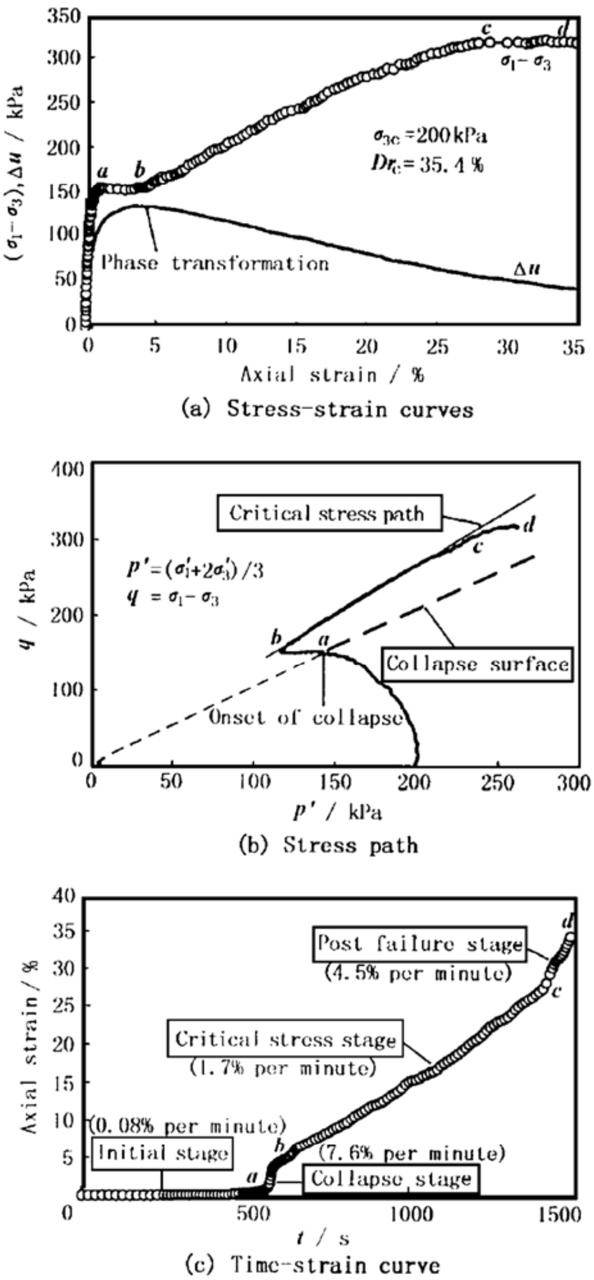

Fig.3 Typical results of undrained test on sand with initial state above and near steady state line (sample 2))

It is of interest to note that the collapse of the sample did not cause any decrease in the deviator stress, even though the pore pressure increased rapidly during collapse. As a result, the stress path in the collapse stage followed a horizontal line until reaching the critical stress path (Fig. 3 (b)). This provides experimental evidence to prove that in a p'-q plot, a collapse surface is a straight line through the origin [9, 10], rather than through the steady state point [11].

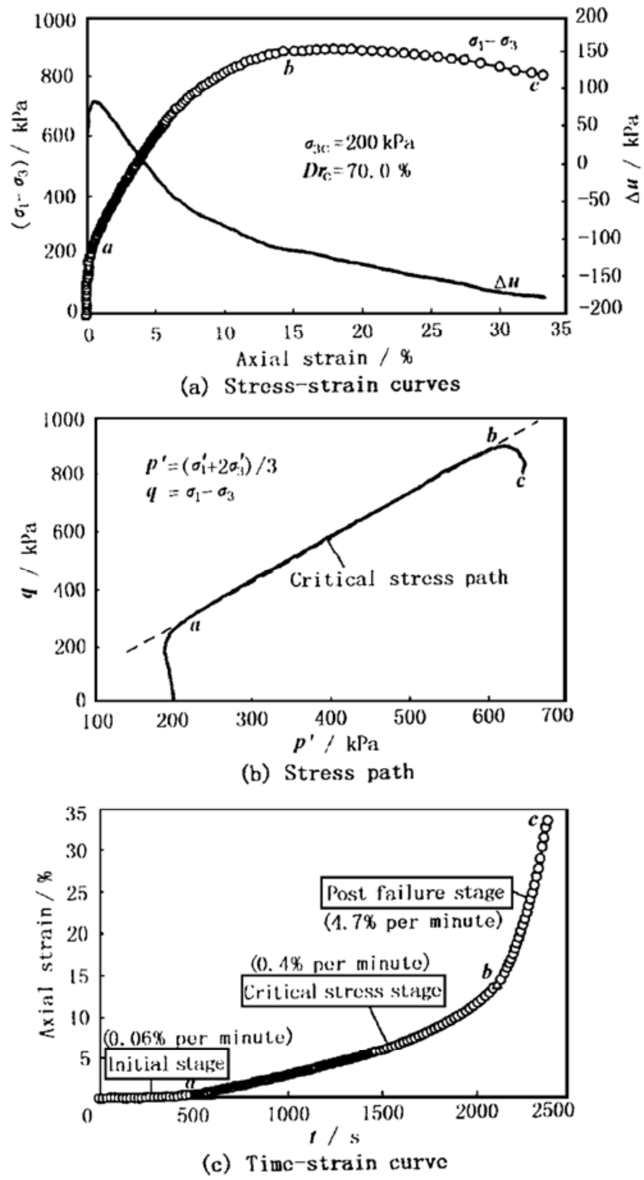

Fig.4 Typical results of undrained test on dense sand (sample ③)

Fig.4 shows the results of the undrained test on sample ③, for which the consolidation state was far below the steady state line (Fig.6). There were three distinct deformation stages in this test: initial stage (from the origin to point a), critical stress stage (from a to b) and post-failure stage (from b to c), in which the strain rates remained constant and are equal to 0.06%, 0.4% and 4.7% per minute, respectively. No collapse stage deformation occurred in this test. In the critical stress stage, the deviator stress increased from 253 kPa at phase transformation to a peak value of 887 kPa at a strain of 15%, accompanied by a decrease in pore pressure from 86 to −114 kPa. The post failure stage deformation started once the deviator stress reached the peak. In this stage, the sample deformed with a relatively fast strain rate of 4.7% per minute (Fig.4 (c)); the deviator stress slightly decreased with further strain, while the pore pressure continuously decreased (Fig.4 (a)); also, the effective stress path moved away from the critical stress path (Fig.4 (b)). These results may indicate that in such a test, shear band may have occurred and particle breakage in the shear zone may become significant in the post-failure stage. No temporary steady state appeared in this test.

4.2 Drained tests

The deformation stages are not so distinct in the drained tests as in the undrained tests. It is seen in Fig.2 that the deviator stress increased with a decrease of void ratio, until a peak was reached at a stain of 12.5%; subsequently, the deviator stress remained constant with further strain, while the void ratio remained constant only up to a strain of 17% and then increased continuously until the end of the test. The strain rate increased until the peak, and then remained constant (17.3% per minute). A temporary steady state was reached between strains of 12.5% and 17%.

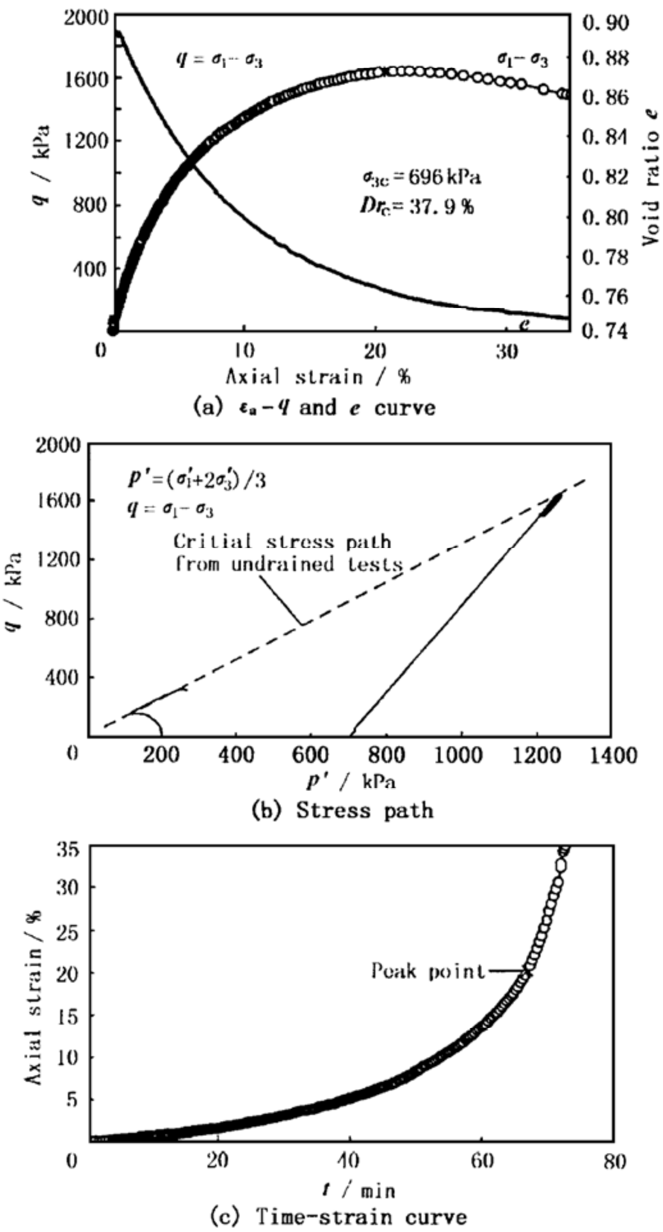

Fig.5 Results of drained tests on sand under high effective consolidation stress (sample ⑤)

It is seen in Fig.5 that in the drained test on sample (5), the deviator stress continuously increased until reaching a peak value, then decreased slightly with further strain; while the void ratio continuously decreased until the end of the test (Fig.5 (a)). When the deviator stress reached the peak value, the effective stress path also reached the critical stress path which was determined from the undrained tests, and then moved down with further strain (Fig. 5 (b)). The strain rate also remained constant after the peak, as in sample (4). A shear band might developed after the peak, and no steady state of deformation could be determined from the test.

5 Modified definition of the steady state

Casagrande [2] defined that the critical void ratio is one at which a cohesionless soil can undergo any amount of deformation without volume change under drained condition. The critical void ratio is a function of the effective stress, and can be reached from either a loose or dense state in drained tests [3]. Taylor [4] supplemented that under undrained condition, the critical void ratio is one at which shearing leads to no strength change.

Two further concepts, the critical state and the steady state of deformation were developed, from the concept of the critical void ratio, by Roscoe et al [5] and Poulos [6], respectively. Roscoe et al [5] stated that the critical state is an ultimate state at which any further increment of shear distortion will not result in change of void ratio under drained condition, or change of effective stress under undrained condition. The critical state points so defined can be expected to be located on or near a line on a yield surface. Poulos [6] stated that the steady state of deformation for any mass of particles is that state in which the mass is continuously deforming at constant volume, constant normal effective stress, constant shear stress, and constant velocity. The steady state of deformation is achieved only after all particle orientation has reached a statistically steady state condition and after all particle breakage, if any, is complete, so that the shear stress needed to continue deformations and the velocity of deformation remain constant.

It is observed that the definition of the critical state combines Casagrande's and Taylor's definitions, and implies that the critical state points are located on a unique line. The definition of the steady state emphasizes that it is a continuous deformation state with constant parameters. Despite their different statements, both concepts in fact describe the same failure phenomenon that a soil mass is continuously deforming at constant volume and constant stresses [1, 16~18]. Only the term "steady state" is used in this paper.

The definition of the steady state of deformation appears not to be completely valid in the case of the quasi-steady state behavior. The definitions state that the steady state is a unique ultimate state, and should be achieved at large strain. Experimental data indicates the temporary steady state or quasi-steady state satisfies the requirements stated in the definitions of the steady state; however, after phase transformation, the deviator stress may increase continuously with further strain, or the "second steady state" occurs at very large axial strain (e.g. Fig.1). It is thus considered that for sands with QSS behavior, the achievable strain in triaxial apparatus might not be large enough for the ultimate steady state to be reached [6], or the second steady state should be the "real" steady state [7]. Therefore, the above definitions of steady state are ambiguous regarding which steady state should be used in design.

The results of triaxial tests on sands are significantly affected by some factors including end restraint [8]. The second steady state at very large strain, as shown in Fig.1 and in the tests by Verdugo and Ishihara [7], should be the state at which both the state of a sand sample and the effect of end restraint have reached ultimate states. Thus, the temporary or quasi-steady state, which occurs during collapes, may be the steady state in the QSS behavior.

Desrues et al [9] found, using computed tomography technique to measure local void ratio, that for a dense sample with void ratio less than critical, obvious shear bands can occur and the void ratio in the shear zone is larger than the global void ratio of the sample. The shear strength of dense sands may thus be underestimated due to the formation of the shear band, since the shear strength is significant affected by the void ratio in shear zone. This may be the reason why a continuous decrease of pore pressure at large strain can be accompanied by a decrease of deviator stress in an undrained test (Fig. 4), or a continuous decrease of global void ratio can also be accompanied by a decrease of deviator stress in a drained test (Fig. 5); and the steady state points from drained tests on dense sample are always lower than those from the tests on loose samples [7].

As stated by Poulos et al [2], the steady state strength is the fundamental strength of a sand for a given void ratio, which can not decrease as a peak strength does, or increase after all particle orientation or (and) all particle breakage is complete. However, the steady state strength measured in laboratory tests may be underestimated due to the formation of shear band, or overestimated due to the effect of end restraint. The definition of steady state should avoid confusion which may result from the conventional triaxial tests on sands, and a modified definition of steady state is therefore suggested as follows:

The steady state of deformation for any mass of particles is that state in which the mass is continuously deforming at constant volume, constant normal effective stress, constant shear stress, and constant velocity. The constant shear stress is the minimum strength of the mass, which is dependent on the local void ratio within the shear zone.

6 Steady state strength of Unimin sand

All undrained triaxial tests on loose Unimin sand samples, among which the lowest relative density at consolidation stage was equal to 14%, showed the quasi-steady state behavior in which a rapid collapse deformation was followed by a continuous increase of deviator stress (Figures 1 and 2). The steady state points of these tests are shown in

Fig. 6. It can be seen that all the steady state points are located on or near a straight line in an e-$\log p'$ plot, except the two points from dense samples in which no steady state could be determined from the tests.

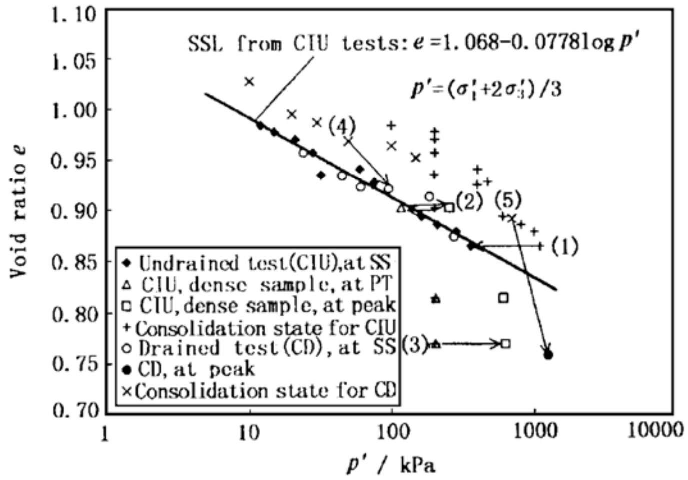

Fig. 6　Summary of triaxial tests on Unimin sand

The steady state points from drained tests also reside on or near the steady state line obtained from the undrained tests (Fig. 6). This indicates that the steady state strength for a given void ratio from drained and undrained tests are the same for Unimin sand. Similar results were observed for Syncrude tailings[5], Erksak sand[16], Toyoura sand[7]. Conversely, based on the test data from Castro[1], Alarcon et al[20] found a significant difference between the steady state lines from drained and undrained triaxial tests. However, among the three sands used by Castro, only in sand B there were six drained steady state points locating above the undrained steady state line with two drained points lying on the undrained line (Fig. 70 in literature [1]). For the two other sands, the undrained and drained steady state points located on the same steady state lines (Figs. 90 and 111 in literature [1]).

7　Conclusions

(1) All loose Unimin sand samples exhibit the quasi-steady state behavior in undrained triaxial stress-controlled tests. In such a behavior, there are usually four distinct deformation stages: initial stage, collapse stage, critical stress stage and post failure stage, and in each of them the strain rate remains constant. The steady state, in which the shear stress, effective stresses, pore pressure and strain rate remain constant, can be reached in the collapse stage. In the critical stress stage, the state points follow the critical stress path of the sand, which is defined as the locus of the stress states when the friction resistance of a soil mass is fully mobilized under given effective stresses.

(2) In drained triaxial tests on Unimin sand, the strain rate remains constant after a peak resistance is reached, and the steady state may occur only after the peak.

(3) The modified definition of the steady state of deformation is suggested to avoid the potential confusion due to the limitation of the triaxial tests.

(4) The steady state points of Unimin sand located on or near a straight line on an e-$\log p'$ plot, regardless of the drainage conditions

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