

DOI: 10.11779/CJGE20240531

竖向振动基桩周围饱和土动态响应研究

付鹏程^{1,2}, 吴君涛^{*1,2}, 易恩泽^{1,2}, 王奎华^{1,2}, 张智卿³, 汤旅军⁴

(1. 浙江大学建筑工程学院, 浙江 杭州 310058; 2. 浙江大学平衡建筑研究中心, 浙江 杭州 310063; 3. 温州理工学院建筑与能源工程学院, 浙江 温州 325035; 4. 浙江水利水电学院建工学院, 浙江 杭州 310018)

摘要: 基于 Biot 饱和多孔介质理论建立竖向激励作用下桩-饱和土耦合振动模型, 研究三维轴对称条件下桩周饱和土振动响应规律。通过引入势函数并采用算子分解法及分离变量法, 结合桩土耦合振动边界条件, 求解得到饱和土与超静孔隙水压动态响应解析解。在退化工况下, 解析解计算结果与既有桩周弹性介质土的动力响应结果几乎一致。基于经验证后的解析解, 开展弹性波在完全饱和土中传播机理的研究。通过场地土位置-时间-响应关系, 可明确观察到饱和土中存在的快纵波(P_1 波)与剪切波(S 波)。对不同工况下的桩-饱和土耦合系统进行参数分析, 得到桩周饱和土的波动响应规律, 其能够对基础减隔振以及桩基工程检测提供工程指导以及理论支撑。

关键词: 桩基础; 饱和土; 动力响应; 解析解; 波动规律

中图分类号: TU473.1 文献标识码: A 文章编号: 1000-4548(2025)10-2215-09

作者简介:付鹏程(2001—),男,博士研究生,主要从事桩基动力理论研究及测试技术开发工作。E-mail: fpengc@163.com。

Study on dynamic response of saturated soil around piles under vertical vibration

FU Pengcheng^{1,2}, WU Juntao^{1,2}, YI Enze^{1,2}, WANG Kuihua^{1,2}, ZHANG Zhiqing³, TANG Lüjun⁴

(1. College of Civil Engineering and Architecture, Zhejiang University, Hangzhou 310058, China; 2. Center for Balance Architecture, Zhejiang University, Hangzhou 310063, China; 3. College of Architecture and Energy Engineering, Wenzhou University of Technology, Wenzhou 325035, China; 4. College of Civil Engineering and Architecture, Zhejiang University of Water Resources and Electric Power, Hangzhou 310018, China)

Abstract: A coupled pile-saturated soil model is proposed to investigate the response of saturated soil under three-dimensional axisymmetric conditions based on Biot's theory of saturated porous media. Combined with the pile-soil coupling vibration boundary conditions, an analytical solution in the frequency domain for the saturated soil and the resulting pore water pressure is derived. This is achieved through the introduction of potential functions and the application of operator decomposition and separation of variables methods. The derived solutions are compared with existing dynamic response results for elastic soil surrounding a vibrating pile. The wave propagation mechanism in fully saturated soil is investigated based on an analytical solution which has been verified. The analysis result reveals the presence of fast longitudinal waves (P_1 waves) and shear waves (S waves), as evidenced by three-dimensional position-time-response plots. Further parameter analysis under varying operational conditions elucidates the fluctuation response characteristics of saturated soil around the pile. The findings provide valuable insights and have significant implications for engineering practice.

Key words: pile foundation; saturated soil; dynamic response; analytic solution; wave propagation rules

0 引言

桩基础作为建筑工程中重要的基础形式, 广泛应用于高层建筑、桥梁和港口等大型结构物中。在复杂

地质条件下, 桩基础能够有效地将上部结构的荷载传递至地基土中, 其可靠性和稳定性直接关系到整个工程的安全性和使用寿命。因此, 研究桩土相互作用具

基金项目: 浙江省自然科学基金探索项目(LTGG24E080001); 国家自然科学基金项目(52108349, 52178358, 52178367)This work was supported by the Natural Science Foundation of Zhejiang Province (Grant No. LTGG24E080001) and National Natural Science Foundation of China (Grant Nos. 52108349, 52178358, 52178367).

收稿日期(Received): 2024-05-31 修回日期(Revised): 2024-11-10 接受日期(Accepted): 2024-12-06 网络出版日期(Online): 2024-12-07

*通信作者 (E-mail: wujuntao31@126.com)

有重要的工程实际意义。

建立桩土耦合振动模型的重难点之一在于对桩周土体的模拟。近年来,国内外学者已经对不同土体模型下的桩基振动响应进行了长足的研究,并取得了显著成果^[1-3]。基于动力 Winkler 模型,王奎华等^[4]通过积分变换推导了有限长桩的竖向振动时域解析解;基于平面应变模型,胡昌斌等^[5]建立了考虑土层之间相互作用的成层土模型,并用于指导基桩动力检测;基于三维连续介质模型,丁选明等^[6]、郑长杰等^[7]引入势函数进行土体振动解耦,利用桩土完全耦合的条件得到桩振动响应解。在此基础上,吴文兵等^[8]通过虚土桩模型模拟黏弹性支撑和固定支撑,研究了桩底土材料参数对单桩动力响应的影响。

尽管已经积累了大量关于桩基振动响应的研究成果,但针对桩周土体振动的研究相对较少。实际上,桩基振动不仅影响自身结构,还会对场地土体产生显著影响,土体的振动特性在一定程度上决定了地基的整体动力性能和工程结构的抗震性能^[9]。考虑桩土相互作用下的土体振动响应能够为建筑物的减隔振设计提供更为全面的理论依据^[10]。此外,研究桩周土体的振动规律同样可以为桩基完整性测试,例如旁孔透射波法检测技术提供理论指导^[11]。Wu 等^[12-13]对竖向振动桩基周围场地的动态响应进行了理论分析,研究了成层黏弹性土以及均质饱和土环境下的桩土相互作用。虽然已有许多学者从不同角度对饱和土中的桩基振动响应开展了大量研究,包括竖向荷载作用下端承桩^[14]、摩擦桩^[15]的桩顶竖向响应和横向荷载作用下的桩基振动响应^[16-17],但大多数仍集中于基桩自身振动特性,对于桩周饱和土在基桩竖向振动下的动力响应的理论研究尚不完备,桩周饱和土中波的传播规律也有待进一步探索。

基于以上背景,本文将从 Biot 建立的土-孔隙水两相介质相互作用的控制方程出发,深入探讨竖向基桩振动下桩周饱和土的振动响应特性,重点分析土体作为双相介质材料在桩基振动下的波动规律,以期为工程实践提供有价值的理论支持和应用参考。

1 计算模型以及定解问题

1.1 弹性支承桩-桩周饱和土简化模型

本节将介绍饱和土层中弹性支承桩受竖向激振作用时的桩土竖向耦合振动问题,计算简图如图 1 所示。图中,桩长为 h_p , 桩径为 r_p 。为简便起见,在柱坐标下进行计算,并引入如下假设:

(1) 桩身为圆柱形的均质弹性材料,可视为一维弹性杆件,底部弹性支承。

(2) 桩周饱和土为均质且各向同性的两相多孔弹

性介质,满足 Biot 多孔介质理论。

(3) 桩与土在接触面上无相对滑动,满足力平衡与位移协调条件。

(4) 桩顶受到竖向激振力 $q = Q e^{i\omega t}$, 其中 i 是虚数, ω 是振动圆频率, t 是时间变量。

(5) 桩与饱和土的振动均满足小变形条件且在桩顶动荷载作用下作小变形谐和振动。

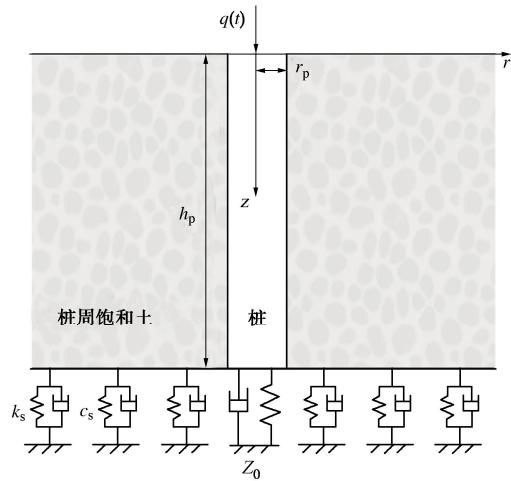


图 1 桩-饱和土竖向耦合振动计算简图

Fig. 1 Schematic of pile-saturated soil coupled vibration model

1.2 控制方程

(1) 饱和介质动力平衡方程

根据 Biot 波动方程^[18-19],对于轴对称问题,任意位置处土单元体沿径向与竖向的动力平衡方程为

$$\begin{aligned} \mu_s \nabla^2 u_r + (\lambda_c + \mu_s) \frac{\partial e}{\partial r} - \mu_s \frac{u_r}{r^2} - \alpha M \frac{\partial \zeta_d}{\partial r} \\ = \rho \frac{\partial^2 u_r}{\partial t^2} + \rho_f \frac{\partial^2 w_r}{\partial t^2}, \end{aligned} \quad (1a)$$

$$\begin{aligned} \mu_s \nabla^2 u_z + (\lambda_c + \mu_s) \frac{\partial e}{\partial z} - \alpha M \frac{\partial \zeta_d}{\partial z} \\ = \rho \frac{\partial^2 u_z}{\partial t^2} + \rho_f \frac{\partial^2 w_z}{\partial t^2}, \end{aligned} \quad (1b)$$

$$\begin{aligned} \alpha M \frac{\partial e}{\partial r} - M \frac{\partial \zeta_d}{\partial r} \\ = \rho_f \frac{\partial^2 u_r}{\partial t^2} + m \frac{\partial^2 w_r}{\partial t^2} + b \frac{\partial w_r}{\partial t}, \end{aligned} \quad (1c)$$

$$\begin{aligned} \alpha M \frac{\partial e}{\partial z} - M \frac{\partial \zeta_d}{\partial z} \\ = \rho_f \frac{\partial^2 u_z}{\partial t^2} + m \frac{\partial^2 w_z}{\partial t^2} + b \frac{\partial w_z}{\partial t}. \end{aligned} \quad (1d)$$

式中: $u(r,z,t)$, $w(r,z,t)$ 分别为土骨架与孔隙流体的位移; ρ 与 ρ_f 分别为土体总密度与孔隙流体密度; λ_c 与 μ_s 为考虑土体材料滞回阻尼 D_s ^[5] 的 Lame 常数; α 与 M 分别为 Biot 系数与 Biot 模量; p_f 为超静孔隙水压;

$\lambda_e = \lambda_s + \alpha^2 M$; $m = \rho_f/n$; $b = \eta/k$ 为介质耗散参数, 表征土骨架与孔隙流体之间由相对运动而产生摩擦损耗, η 为流体黏滞系数, k 为渗透性系数, 一般而言砂土的 b 值较小, 黏性土的 b 值较大; e , ζ_d 以及 p_f 的表达式详见附录。

(2) 桩基动力平衡方程

设桩身坚向振动位移为 $u_p(z,t)$, 在小变形假设下忽略桩身坚向振动时产生的径向位移, 可列桩身微单元段的动力平衡方程:

$$E_p S_p \frac{\partial^2 u_p}{\partial z^2} + f_s = \rho_p S_p \frac{\partial^2 u_p}{\partial t^2} \quad . \quad (2)$$

式中: E_p , S_p 以及 ρ_p 分别为桩身弹性模量、横截面积以及密度; $f_s(z,t)$ 为桩周饱和土对桩身微单元段作用的侧摩阻力。

1.3 定解条件

$$u_r(\infty, z, t) = 0 \quad , \quad (3a)$$

$$\sigma_z(r, 0, t) = 0 \quad , \quad (3b)$$

$$u_z(r_p, z, t) = u_p(z, t) \quad , \quad (3c)$$

$$u_r(r_p, z, t) = 0 \quad , \quad (3d)$$

$$w_r(r_p, z, t) = 0 \quad , \quad (3e)$$

$$2\pi r_p \cdot \tau(r_p, z, t) = -f_s(z, t) \quad , \quad (3f)$$

$$E_s \frac{\partial u_z}{\partial z} + k_s u_z + c_s \frac{\partial u_z}{\partial t} \Big|_{z=h_p} = 0 \quad , \quad (3g)$$

$$E_p \frac{\partial u_p}{\partial z} + Z_0 u_z \Big|_{z=h_p} = 0 \quad , \quad (3h)$$

$$E_p \frac{\partial u_p}{\partial z} + q(t) \Big|_{z=0} = 0 \quad . \quad (3i)$$

式中: E_s 为桩底土弹性模量; k_s 与 c_s 为桩底土与桩周土之间均布 Viogt 体的弹性系数以及黏性系数; Z_0 为桩底部弹性支承阻抗^[20]; σ_z 与 τ 表达式详见附录。

2 桩土耦合振动问题求解

基于简化假设 (5), 令:

$$\left. \begin{aligned} u_p &= U_p(z) \cdot e^{i\omega t}, \\ u_r &= U_r(r, z) \cdot e^{i\omega t}, \quad u_z = U_z(r, z) \cdot e^{i\omega t}, \\ w_r &= W_r(r, z) \cdot e^{i\omega t}, \quad w_z = W_z(r, z) \cdot e^{i\omega t}, \\ f_s &= F_s(z) \cdot e^{i\omega t}, \quad p_f = P_f(r, z) \cdot e^{i\omega t} \end{aligned} \right\} \quad (4)$$

式中: U , W 为土骨架与孔隙水的位移频域幅值; U_p 为桩身质点坚向位移的频域幅值; F_s 为桩侧摩阻力频域幅值; P_f 为孔隙水压力频域幅值。

2.1 饱和土层振动响应求解

式 (1) 中, 土骨架位移与孔隙流体位移相互耦合。对固相位移与液相位移分别引入势函数进行解耦^[14]:

$$\left. \begin{aligned} U_r &= \frac{\partial \phi_s}{\partial r} + \frac{\partial^2 \psi_s}{\partial z \partial r}, \quad U_z = \frac{\partial \phi_s}{\partial z} - \frac{1}{r \partial r} \left(r \frac{\partial \psi_s}{\partial r} \right) \quad , \\ W_r &= \frac{\partial \phi_f}{\partial r} + \frac{\partial^2 \psi_f}{\partial z \partial r}, \quad W_z = \frac{\partial \phi_f}{\partial z} - \frac{1}{r \partial r} \left(r \frac{\partial \psi_f}{\partial r} \right) \quad . \end{aligned} \right\} \quad (5)$$

联立式 (1) 以及边界条件式 (3a), 由 Helmholtz 矢量分解定理, 并引入分离常数 λ 可得

$$\begin{aligned} \phi_s &= K_0(\xi_1 r) [C_{s1} \sin(\lambda z) + D_{s1} \cos(\lambda z)] + \\ &\quad K_0(\xi_2 r) [C_{s2} \sin(\lambda z) + D_{s2} \cos(\lambda z)] \quad , \end{aligned} \quad (6a)$$

$$\begin{aligned} \phi_f &= K_0(\xi_1 r) [C_{f1} \sin(\lambda z) + D_{f1} \cos(\lambda z)] + \\ &\quad K_0(\xi_2 r) [C_{f2} \sin(\lambda z) + D_{f2} \cos(\lambda z)] \quad , \end{aligned} \quad (6b)$$

$$\psi_s = K_0(\zeta r) [A_s \sin(\lambda z) + B_s \cos(\lambda z)] \quad , \quad (6c)$$

$$\psi_f = K_0(\zeta r) [A_f \sin(\lambda z) + B_f \cos(\lambda z)] \quad . \quad (6d)$$

式中: $C_{s1,2}$, $D_{s1,2}$, $C_{f1,2}$, $D_{f1,2}$, $A_{s,f}$ 以及 $B_{s,f}$ 为待定系数, 需根据边界条件求解; ξ 与 ζ 的表达式详见附录。考虑微分算子方程的相关性, 联立边界条件式 (3b), (3d), (3e), 经计算可得土骨架位移、孔隙流体位移以及超静孔隙水压表达式:

$$U_r = \sum_{n=1}^{\infty} C_{s1n} \sin(\lambda_n z) [-\xi_{1n} K_1(\xi_{1n} r) - \eta_{1n} \xi_{2n} K_1(\xi_{2n} r) + \eta_{2n} \zeta_n \lambda_n K_1(\zeta_n r)] \quad , \quad (7a)$$

$$U_z = \sum_{n=1}^{\infty} C_{s1n} \cos(\lambda_n z) [\lambda_n K_0(\xi_{1n} r) + \eta_{1n} \lambda_n K_0(\xi_{2n} r) - \eta_{2n} (\zeta_n)^2 K_0(\zeta_n r)] \quad , \quad (7b)$$

$$W_r = \sum_{n=1}^{\infty} C_{s1n} \sin(\lambda_n z) [-g_1 \xi_{1n} K_1(\xi_{1n} r) - g_2 \eta_{1n} \xi_{2n} K_1(\xi_{2n} r) + g_3 \eta_{2n} \zeta_n \lambda_n K_1(\zeta_n r)] \quad , \quad (7c)$$

$$W_z = \sum_{n=1}^{\infty} C_{s1n} \cos(\lambda_n z) [g_1 \lambda_n K_0(\xi_{1n} r) + g_2 \eta_{1n} \lambda_n K_0(\xi_{2n} r) - g_3 \eta_{2n} (\zeta_n)^2 K_0(\zeta_n r)] \quad , \quad (7d)$$

$$P_f = -M \sum_{n=1}^{\infty} C_{s1n} \sin(\lambda_n z) [(\alpha + g_1)(\beta_1)^2 \cdot K_0(\xi_{1n} r) + \eta_{1n} (\alpha + g_2)(\beta_2)^2 K_0(\xi_{2n} r)] \quad . \quad (7e)$$

式中: g_i ($i=1,2,3$) 与 η_j ($j=1,2$) 的表达式详见附录; λ_n ($n=1,2,3,\dots$) 为超越方程 $\lambda_n \tan(\lambda_n h_p) = (k_s + i\omega c_s)/E_s$ 的各阶特征根, 可以通过计算机求得, 在后续推导中可视为已知参量。其余特征根 ξ_{1n} , ξ_{2n} 以及 ζ_n 可通过 λ_n 表示。唯一待定系数 C_{s1n} 须利用桩土耦合条件结合桩身坚向振动进行求解。在此之前, 需计算桩土间的相互作用力。

2.2 桩周土剪切刚度计算

联立式 (7b) 以及边界条件式 (3c), (3f), 根据定义, 桩侧壁处单位桩长内桩周饱和土的各阶剪切刚度为

$$k_n = \frac{F_s(z)}{U_z \Big|_{r=r_p}} = 2\pi r_p \mu_s \frac{C_{k1}}{C_{k2}} \quad . \quad (8)$$

式中: C_{k1} 与 C_{k2} 表达式详见附录。桩身任意深度处受到来自桩周土体的摩擦力大小为

$$F_s(z) = -k U_z(r_p, z) = -\sum_{n=1}^{\infty} k_n U_n \cos(\lambda_n z) \quad . \quad (9)$$

负号表示与桩身质点运动方向相反, 式中 U_n 表达式为

$$U_n = C_{s1n} \left[\lambda_n K_0(\xi_{1n} r_p) + \eta_{1n} \lambda_n K_0(\xi_{2n} r_p) - \eta_{2n} (\zeta_n)^2 K_0(\zeta_n r_p) \right] \quad . \quad (10)$$

2.3 桩顶位移导纳函数求解

得到桩周土剪切刚度后, 结合桩顶与桩底的边界条件, 可以进行桩身竖向振动响应求解。将式(10)代入将桩身竖向振动控制方程(2), 整理后有

$$(C_p)^2 \frac{d^2 U_p}{dz^2} + \omega^2 U_p = \frac{1}{\rho_p S_p} \sum_{n=1}^{\infty} k_n U_n \cos(\lambda_n z) \quad . \quad (11)$$

式中: $C_p = (E_p / \rho_p)^{1/2}$ 为桩身一维弹性波波速。求解二阶非齐次微分方程(11), 并使用三角函数的正交性对各阶特征根进行解耦, 可得桩身竖向振动的完整解析表达式为

$$U_p(z) = A_0 M(z) + B_0 N(z) \quad . \quad (12)$$

其中:

$$M(z) = \sin \left(\frac{\omega}{C_p} z \right) + \sum_{n=1}^{\infty} C_n \frac{F_{1n}}{F_{3n}} \cos(\lambda_n z) \quad , \quad (13a)$$

$$N(z) = \cos \left(\frac{\omega}{C_p} z \right) + \sum_{n=1}^{\infty} C_n \frac{F_{2n}}{F_{3n}} \cos(\lambda_n z) \quad , \quad (13b)$$

$$F_{1n} A_0 + F_{2n} B_0 = F_{3n} U_n \quad . \quad (13c)$$

式中: 各阶待定系数 C_n , F_{1n} , F_{2n} 以及 F_{3n} 的表达式详见附录。

令 $m(z) = dM(z)/dz$, $n(z) = dN(z)/dz$, 将式(12)代入边界条件式(3h)有

$$\frac{A_0}{B_0} = -\frac{E_p n(h_p) + Z_0 N(h_p)}{E_p m(h_p) + Z_0 M(h_p)} = \delta \quad . \quad (14)$$

根据边界条件式(3i), 联立式(14)有

$$A_0 = -\frac{C_p}{\omega E_p} Q(\omega), \quad B_0 = -\frac{C_p}{\delta \omega E_p} Q(\omega) \quad . \quad (15)$$

至此, 定解问题中所有待定系数均已求解完成。根据定义, 桩顶位移导纳函数为

$$H_{p0}(\omega) = \frac{U_p(0)}{Q(\omega)} = -\frac{C_p}{E_p \delta \omega} \cdot \left[1 + \delta \sum_{n=1}^{\infty} C_n \frac{F_{1n}}{F_{3n}} + \sum_{n=1}^{\infty} C_n \frac{F_{2n}}{F_{3n}} \right] \quad . \quad (16)$$

2.4 饱和土与超静孔隙水压时域响应半解析解

由式(7)可知, 桩周饱和土与超静孔隙水压动力响应的唯一待定系数为 C_{sln} 。联立式(12), (13), (15), 可以求得各阶的待定系数 C_{sln} 的表达式:

$$C_{sln} = \chi_n Q(\omega) \quad . \quad (17)$$

式中: χ_n 表达式详见附录。将表达式代回式(7)可得频域内土骨架和孔隙流体的任意一点在竖向和径向的位移幅值以及超静孔隙水压力的幅值。通过对导纳函数与激振力之积作傅里叶逆变换, 可得时域内饱和土任意一点位移与超静孔隙水压的半解析解。

3 解的合理性验证

目前关于竖向振动桩周饱和土动力响应的研究较少, 为验证本文解析解的合理性, 将本文退化解结果与振动桩周弹性土的动力响应结果^[11]进行对比。使用半正弦形式的激振力模拟桩顶均布荷载, 即

$$q(t) = \begin{cases} q_{\max} \sin(\theta t) & \left(0 < t \leq \frac{\pi}{\theta} \right) \\ 0 & \left(t > \frac{\pi}{\theta} \right) \end{cases} \quad . \quad (18)$$

式中: θ 为振动圆频率; q_{\max} 为激振幅值。桩底部弹性支承阻抗 Z_0 的取值为^[20]

$$Z_0 = K_0 + i C_0 = \frac{4\mu_{sb}}{\pi(1-\nu_{sb})r_p} + i \frac{3.2\mu_{sb}}{\pi(1-\nu_{sb})\nu_{sb}} \quad . \quad (19)$$

式中: μ_{sb} , ν_{sb} 分别为桩底土的剪切模量与泊松比。桩底土与桩周土的接触面上均有 Kelvin 元件, 相关力学参数取为^[5]

$$k_s = E_{sb}, \quad c_s = 10^4 \quad . \quad (20)$$

式中: k_s 与 c_s 分别为 Kelvin 元件的弹性系数与黏性系数; E_{sb} 为桩底土的弹性模量。

为便于比较不同的动力响应计算结果, 引入如下无量纲量^[21]: $\lambda^* = \frac{\lambda}{\mu}$, $M^* = \frac{M}{\mu}$, $\rho^* = \frac{\rho_f}{\rho}$, $m^* = \frac{\rho^*}{n}$,

$$b^* = b \left(\frac{r_p}{\sqrt{\mu\rho}} \right), \quad E^* = \frac{E_p}{E_s}, \quad r^* = \frac{r_p}{h_p}, \quad t^* = t \frac{C_p}{h_p}, \quad v_{ur}^* =$$

$$v_{uz} \frac{E_p}{q_{\max} C_p}, \quad v_{uz}^* = v_{uz} \frac{E_p}{q_{\max} C_p}, \quad v_{wr}^* = v_{wr} \frac{E_p}{q_{\max} C_p}, \quad v_{wz}^* =$$

$$v_{wz} \frac{E_p}{q_{\max} C_p}, \quad p_f^* = \frac{p_f}{q_{\max}}$$

将 M^* , ρ^* , m^* , b^* 以及 α 设为无穷小量(例如 10^{-5})后, 桩周双相饱和介质退化为单相介质^[17]。调整其余参数使本文工况与吴君涛等^[11]一致, 并将本文退化解与其计算结果进行对比。不同深度与径向位置处土体

竖向速度响应为如图2所示。

图2(a), (b)展示了不同径向位置处土体坚向振动响应随深度的变化关系。从图2中结果可知, 本文退化解与吴君涛解^[11]一致性良好, 证明了本模型的合理性。

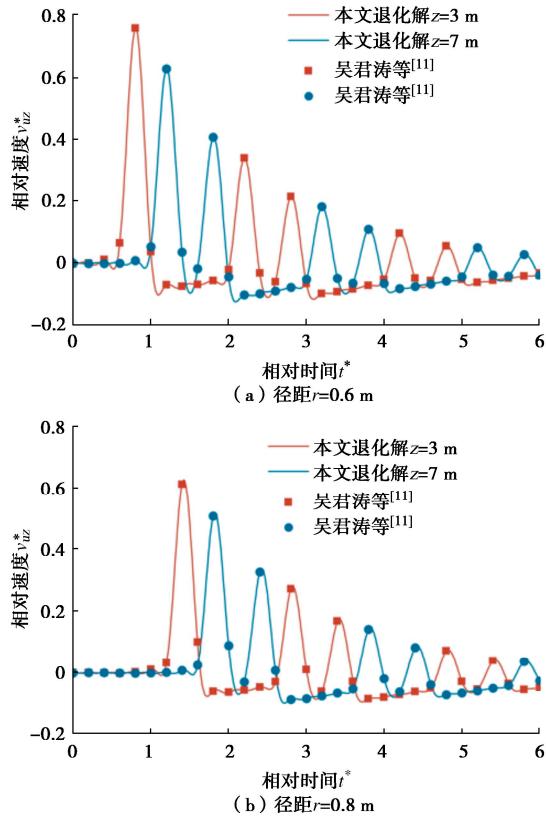


图2 不同位置处土体坚向速度响应对比

Fig. 2 Comparison of vertical velocity response of soil at different locations

4 参数分析

本文模型的合理性已得到验证, 下面将进行模型的参数分析以研究桩周饱和土在不同工况下的振动响应规律。若无特殊说明, 后文中参数均按如下取值: $h_p = 10 \text{ m}$, $E_p = 40 \text{ GPa}$, $\rho_p = 2500 \text{ kg/m}^3$, $v_p = 0.15$, $\rho_s = \rho_{sb} = 1800 \text{ kg/m}^3$, $v_s = v_{sb} = 0.3$, $D_s = 0.02$, $E^* = 500$, $r^* = 0.05$, $\lambda^* = 1.5$, $M^* = 12.2$, $\rho^* = 0.53$, $m^* = 1.1$, $b^* = 10$, $r = 0.6 \text{ m}$, $z = 5 \text{ m}$ 。

与单相介质土体模型相比, 将土体考虑为两相介质引入了孔隙水的作用, 因此本章将首先分析超静孔隙水压的影响因素。考虑到表征土体材料渗透性的无量纲参数 b^* 对于饱和土振动响应的影响最为显著^[22], 本节还将探究 b^* 对桩周饱和土坚向与径向振动响应的影响; 随后分析测点位置、桩身长径比、桩土相对刚度以及土体滞回阻尼等因素对桩周饱和土振动的影响, 并将所得规律与吴君涛等^[11]对黏弹性土的研究结果进行对比。

4.1 孔隙水压的影响因素

为研究坚向振动基桩周围饱和土的孔隙水响应在场地土中的分布规律, 图3采用带投影的三维坐标, 表征超静孔隙水压在坚向上随时间的变化。可以看出位置越深, 孔压响应时间越迟, 这与桩身一维弹性波速有关。值得注意的是, 桩顶受坚向激振时地表处同时产生了向下传播的负孔压; 同时在下行波传至桩底发生反射时, 同深度处桩周饱和土中产生了向上传播的正孔压, 其传播速度为饱和介质中快纵波(P_1 波)的波速。对于场地自由上表面而言, 仅当外部激振力作用时土骨架位移梯度为非零值, 进而产生孔隙水压; 考虑到该位置处饱和土正应力为零的边界条件, 当后续上行波传至桩顶时, 饱和土自由表面便不再产生向下传播的超静孔隙水压。对于桩底深度, 考虑土体底部黏弹性支撑的边界条件, 在桩底土的支承力作用下土骨架体应变为非零值, 进而产生孔隙水压, 因此每当下行波传至桩底时, 均会在对应深度桩周饱和土中产生向上传播的孔压。

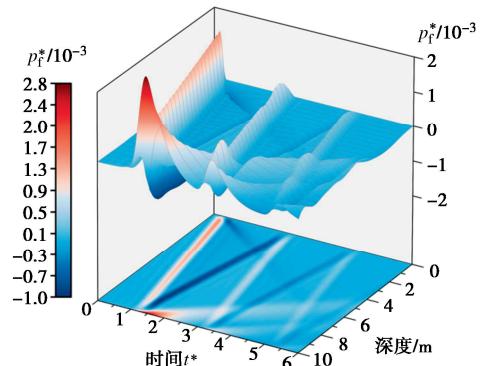


图3 $r=0.6 \text{ m}$ 时桩周饱和土孔压 p_f^* 分布规律

Fig. 3 Distribution pattern of pore pressure p_f^* in saturated soil around pile at $r=0.6 \text{ m}$

实际上, 从饱和土波动方程组出发可以求解出两种纵波(快纵波与慢纵波)与一种剪切波^[19]。其中快纵波便是地震勘探中常被观测到的纵波, 而慢纵波作为一种扩散波, 在低频荷载作用下及易分散和衰减, 对实际工程指导意义不大^[23]。因此后续主要对快纵波与剪切波进行参数分析。

图4展示了不同工况下, $r=0.6 \text{ m}$, $z=5 \text{ m}$ 处的超静孔隙水压响应。图4(a)与图4(b)分别展示了Biot系数 α 以及Biot模量 M^* 对于坚向振动桩周饱和土体超静孔隙水压的影响, 可以看出随着二者的增大, 超静孔隙水压均有所提高。由定义式可知, α 与 M^* 越大表明土颗粒相对于土骨架与孔隙水越难被压缩, 因此, 相同围压下 α 与 M^* 更大的饱和土会产生更大的超静孔隙水压。图4(c)展示了介质耗散参数 b^* 对孔压的影响。随着 b^* 值的增加, 土骨架与孔隙流体之间

更难产生相对位移。在桩周土承受桩基竖向振动荷载作用时,孔隙流体未能及时从被压缩的土骨架中排出,导致超静孔隙水压的上升。图 4 (d) 研究了桩土相对刚度比对孔隙水压的影响,可见饱和土体越“软”,产生的超静孔隙水压幅值越小。

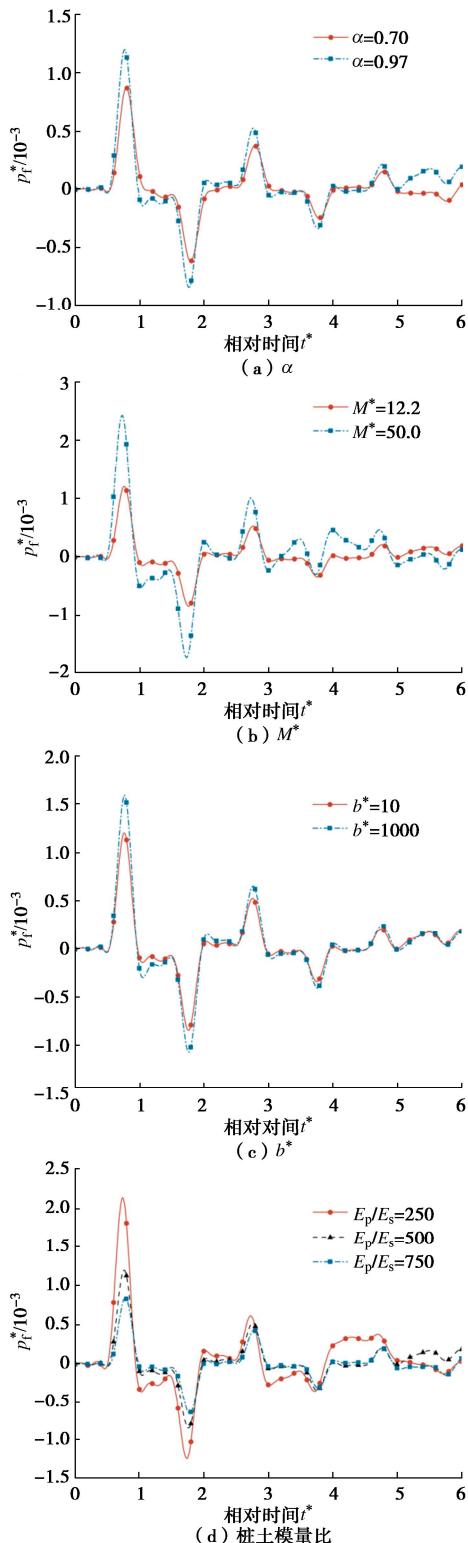


图 4 饱和土体中超静孔隙水压影响因素

Fig. 4 Factors affecting excess water pore pressure in saturated soils

4.2 饱和土波动规律研究

从图 5 可以看出,随着 b^* 的增大,桩周饱和土的竖向响应增大,这可能与超静孔隙水压的影响有关。由图 4 (c) 可知, b^* 值较大的土体中超静孔隙水压力更难消散,在孔压的作用下土体产生了更明显的振动响应。值得注意的是,本文假设桩土接触面无相对位移,因此在地表桩土接触面处的土体竖向响应即为桩顶振动响应。在渗透性较差的土体中孔隙水与土骨架结合更加紧密,相当于增大了桩土相对剪切刚度,因此桩顶的动力响应振幅随着 b^* 值的增加而减小,这与李强等^[14]、Xiao 等^[24]的分析结果一致。

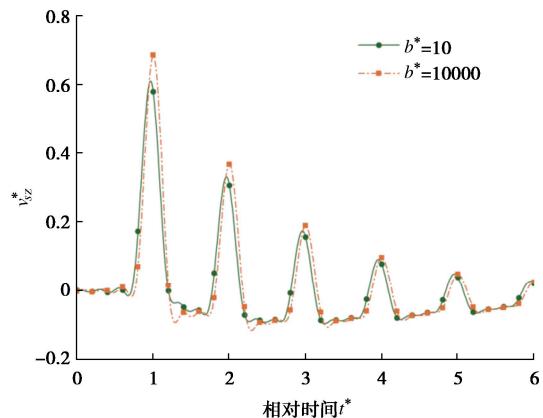


图 5 b^* 对竖向振动桩桩周饱和土竖向振动的影响

Fig. 5 Effect of b^* on vertical vibration of saturated soil around vertical vibrating pile

图 6 展示了桩周饱和土响应点位置对波动规律的影响。对于饱和土竖向振动而言,响应点位置越深、径向距离桩体越远,其首至振动响应的时间越迟、幅值越低。在上、下边界处由于行波的叠加,土骨架竖向响应有明显增大。

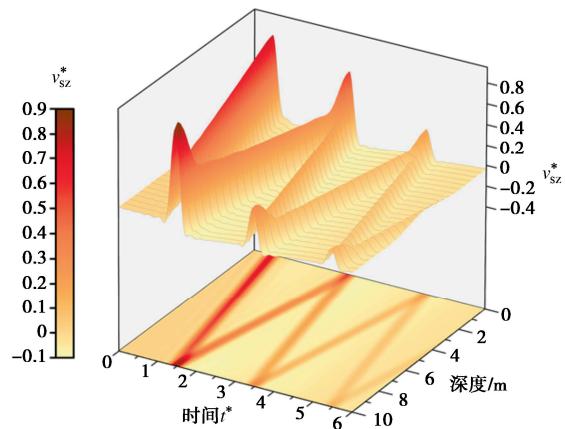


图 6 桩周饱和土土骨架竖向振动响应规律

Fig. 6 Vertical vibration response of saturated soil skeleton around pile

从图 7 可以看出,在竖向振动桩作用下,土体弹性模量增大时饱和土体的响应时间提前,这是因为土

体剪切波速增大。图 8 展示了桩身长径比对桩周饱和土振动的影响, 当桩越“细长”, 饱和土坚向动力响应有所降低。饱和土动态响应还受到土体材料滞回阻尼的作用, 从图 9 可以看出, 土体滞回阻尼对桩周饱和土振动响应有抑制作用。可见桩土相对刚度比、桩身长径比以及土体材料滞回阻尼对饱和土振动影响规律与黏弹性介质土一致^[11]。

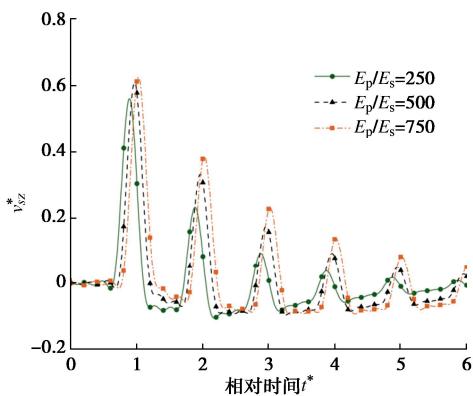


图 7 桩土相对刚度对桩周饱和土竖向振动的影响

Fig. 7 Effect of pile-soil relative stiffness on vertical vibration of saturated soil around vertical vibrating pile

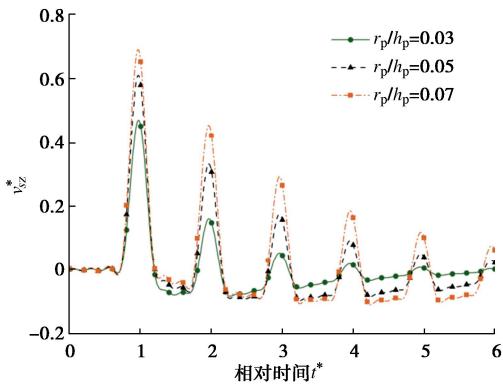


图 8 桩身长径比对桩周饱和土竖向振动的影响

Fig. 8 Effect of pile length-to-diameter ratio on vertical vibration of saturated soil around vertical vibrating pile

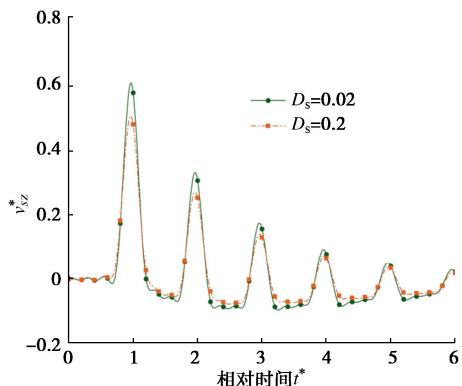


图 9 土体滞回阻尼对竖向振动桩周饱和土竖向振动的影响

Fig. 9 Effect of soil hysteresis damping on vertical vibration of saturated soil around vertical vibrating pile

5 结 论

(1) 本文建立了桩-饱和土耦合模型, 该模型适用于均匀声阻抗完整桩且场地土为均质饱和土的工况条件。基于桩-饱和土耦合模型预测了振动桩周饱和土的动力响应, 分别求得了土骨架、孔隙流体以及超孔隙水压动力响应的解析表达式。

(2) 将本文计算结果与既有桩周单相介质土模型的计算结果进行对比验证。结果表明, 本文退化解计算结果与现有模型的计算结果几乎完全一致。

(3) 基于验证后的解析解, 开展弹性波在完全饱和土中的传播机理研究。通过位置-时间-响应关系, 明确观察到饱和土中存在的快纵波与剪切波。超静孔隙水压通过快纵波传递, 土骨架的坚向位移主要由剪切波引起。

(4) 饱和土中的超静孔隙水压与 Biot 系数 α , Biot 模量 M , 介质损耗参数 b 以及土体弹性模量相关。一般而言, 桩周饱和土压缩性越差、土骨架渗透性越小, 则基桩坚向振动在桩周饱和土中产生的超静孔隙水压越大。

(5) 桩土相对刚度比、桩身长径比以及土体材料滞回阻尼对饱和土振动影响规律与黏弹性介质土一致, 既有单相介质土的部分研究结论仍可适用于完全饱和土的动力响应分析。

利益冲突声明/Conflict of Interests: 所有作者声明不存在利益冲突。All authors disclose no relevant conflict of interest.

作者贡献/Authors' Contributions: 付鹏程、吴君涛和王奎华设计研究内容, 张智卿和汤旅军提供研究指导, 付鹏程完成计算分析, 付鹏程、吴君涛和易恩泽参与论文的写作和修改。所有作者均阅读并同意最终稿件的提交。This study was designed by FU Pengcheng, WU Juntao and WANG Kuihua and supervised by ZHANG Zhiqing and TANG Lujun. The analytical solution was derived by FU Pengcheng. The manuscript was drafted and revised by FU Pengcheng, WU Juntao and YI Enze. All the authors have read the last version of manuscript and consented for submission.

参考文献:

- [1] MUKI R, STERNBERG E. On the diffusion of an axial load from an infinite cylindrical bar embedded in an elastic medium[J]. International Journal of Solids and Structures, 1969, 5(6): 587-605.
- [2] NOVAK M, ABOUL-ELLA F, NOGAMI T. Dynamic soil reactions for plane strain case[J]. Journal of the Engineering

- Mechanics Division, 1978, **104**(4): 953-959.
- [3] NOGAMI T, KONAGAI K. Time domain axial response of dynamically loaded single piles[J]. Journal of Engineering Mechanics, 1986, **112**(11): 1241-1252.
- [4] 王奎华, 谢康和, 曾国熙. 有限长桩受迫振动问题解析解及其应用[J]. 岩土工程学报, 1997, **19**(6): 27-35. (WANG Kuihua, XIE Kanghe, ZENG Guoxi. Analytical solution to vibration of finite length pile under exciting force and its application[J]. Chinese Journal of Geotechnical Engineering, 1997, **19**(6): 27-35. (in Chinese))
- [5] 胡昌斌, 王奎华, 谢康和. 基于平面应变假定基桩振动理论适用性研究[J]. 岩土工程学报, 2003, **25**(5): 595-601. (HU Changbin, WANG Kuihua, XIE Kanghe. Study on theories of axial response of dynamically loaded single piles based on plane strain assumption[J]. Chinese Journal of Geotechnical Engineering, 2003, **25**(5): 595-601. (in Chinese))
- [6] 丁选明, 刘汉龙. 大直径管桩在瞬态集中荷载作用下的振动响应时域解析解[J]. 岩土工程学报, 2013, **35**(6): 1010-1017. (DING Xuanming, LIU Hanlong. Time-domain analytical solution of the vibration response of a large-diameter pipe pile subjected to transient concentrated load[J]. Chinese Journal of Geotechnical Engineering, 2013, **35**(6): 1010-1017. (in Chinese))
- [7] 郑长杰, 丁选明, 刘汉龙, 等. 考虑土体三维波动效应的现浇大直径管桩纵向振动响应解析解[J]. 岩土工程学报, 2013, **35**(12): 2247-2254. (ZHENG Changjie, DING Xuanming, LIU Hanlong, et al. Analytical solution to vertical vibration of cast-in-place concrete large-diameter pipe piles by considering 3D wave effect of soils[J]. Chinese Journal of Geotechnical Engineering, 2013, **35**(12): 2247-2254. (in Chinese))
- [8] 吴文兵, 王奎华, 杨冬英, 等. 成层土中基于虚土桩模型的桩基纵向振动响应[J]. 中国公路学报, 2012, **25**(2): 72-80. (WU Wenbing, WANG Kuihua, YANG Dongying, et al. Longitudinal dynamic response to the pile embedded in layered soil based on fictitious soil pile model[J]. China Journal of Highway and Transport, 2012, **25**(2): 72-80. (in Chinese))
- [9] 郑刚, 霍海峰, 雷华阳, 等. 振动频率对饱和黏土动力特性的影响[J]. 天津大学学报, 2013, **46**(1): 38-43. (ZHENG Gang, HUO Haifeng, LEI Huayang, et al. Contrastive study on the dynamic characteristics of saturated clay in different vibration frequencies[J]. Journal of Tianjin University, 2013, **46**(1): 38-43. (in Chinese))
- [10] 何金辉, 李明广, 陈锦剑. 饱和砂土中高频振动沉桩环境影响机理研究[J]. 地基处理, 2022, **4**(S1): 45-50, 57. (HE Jinhui, LI Mingguang, CHEN Jinjian. Study on the environmental impact mechanism of high-frequency vibratory piles driven in saturated soil[J]. Journal of Ground Improvement, 2022, **4**(S1): 45-50, 57. (in Chinese))
- [11] 吴君涛, 王奎华, 肖偲, 等. 弹性支承桩周围土振动响应解析解及其波动规律研究[J]. 岩石力学与工程学报, 2018, **37**(10): 2384-2393. (WU Juntao, WANG Kuihua, XIAO Si, et al. An analytical solution of dynamic response of soil around elastic bearing pile[J]. Chinese Journal of Rock Mechanics and Engineering, 2018, **37**(10): 2384-2393. (in Chinese))
- [12] WU J T, WANG K H, EL NAGGAR M H. Dynamic soil reactions around pile-fictitious soil pile coupled model and its application in parallel seismic method[J]. Computers and Geotechnics, 2019, **110**: 44-56
- [13] WU J T, EL NAGGAR M H, WANG K H. Dynamic response of poroelastic soil adjacent to an axially vibrating pile[J]. Journal of Engineering Mechanics, 2024, **150**(11): 04024084.
- [14] 李强, 王奎华, 谢康和. 饱和土中端承桩纵向振动特性研究[J]. 力学学报, 2004, **36**(4): 435-442. (LI Qiang, WANG Kuihua, XIE Kanghe. Vertical vibration of an end bearing pile embedded in saturated soil[J]. Chinese Journal of Theoretical and Applied Mechanics, 2004, **36**(4): 435-442. (in Chinese))
- [15] 崔春义, 张石平, 杨刚, 等. 考虑桩底土层波动效应的饱和黏弹性半空间中摩擦桩竖向振动[J]. 岩土工程学报, 2015, **37**(5): 878-892. (CUI Chunyi, ZHANG Shiping, YANG Gang, et al. Vertical vibration of floating piles in saturated viscoelastic half-space considering wave effect of subsoil under pile bottom[J]. Chinese Journal of Geotechnical Engineering, 2015, **37**(5): 878-892. (in Chinese))
- [16] FU P, XIE K H. Lateral vibration of offshore piles considering pile-water interaction[J]. International Journal of Structural Stability and Dynamics, 2019, **19**(12): 1950147.
- [17] ZHENG C J, HE R, KOURETZIS G, et al. Horizontal vibration of a cylindrical rigid foundation embedded in poroelastic half-space[J]. Computers and Geotechnics, 2019, **106**: 296-303.
- [18] BIOT M A. Theory of propagation of elastic waves in a fluid-saturated porous solid I low-frequency range[J]. Acoustical Society of America Journal, 1956, **28**(2): 168-178.

- [19] BIOT M A. Mechanics of deformation and acoustic propagation in porous media[J]. Journal of Applied Physics, 1962, **33**(4): 1482-1498.
- [20] RANDOLPH M F, DEEKS A J. Dynamic and static soil models for axial pile response[C]// Proceedings of the 4th International Conference on Application of Stress-Wave Theory to Piles, The Hague, 1992.
- [21] SENJUNTICHAI T, RAJAPAKSE R K N D. Transient response of a circular cavity in a poroelastic medium[J]. International Journal for Numerical and Analytical Methods in Geomechanics, 1993, **17**(6): 357-383.
- [22] ZENG X, RAJAPAKSE R K N D. Dynamic axial load transfer from elastic bar to poroelastic medium[J]. Journal of Engineering Mechanics, 1999, **125**(9): 1048-1055.
- [23] 牛滨华, 孙春岩. 地震波理论研究进展: 介质模型与地震波传播[J]. 地球物理学进展, 2004, **19**(2): 255-263. (NIU Binhu, SUN Chunyan. Developing theory of propagation of seismic waves: medium model and propagation of seismic waves[J]. Progress in Geophysics, 2004, **19**(2): 255-263. (in Chinese))
- [24] XIAO S, WANG K H, GAO L, et al. Dynamic characteristics of a large-diameter pile in saturated soil and its application[J]. International Journal for Numerical and Analytical Methods in Geomechanics, 2018, **42**(11): 1255-1269.

$$e = \frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{\partial u_z}{\partial z}, \quad (\text{A-1})$$

$$\zeta_d = -\left(\frac{\partial w_r}{\partial r} + \frac{w_r}{r} + \frac{\partial w_z}{\partial z} \right), \quad (\text{A-2})$$

$$p_f = -\alpha M e + M \zeta, \quad (\text{A-3})$$

$$\sigma_z = \lambda_s e + 2\mu_s \frac{\partial u_z}{\partial z} - \alpha p_f, \quad (\text{A-4})$$

$$\tau = \mu_s \left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r} \right), \quad (\text{A-5})$$

$$\xi_{1,2}^2 = \beta_{1,2}^2 + \lambda^2, \quad (\text{A-6})$$

$$\beta_{1,2}^2 = \frac{-\varphi_1 \pm \sqrt{\varphi_1^2 - 4\varphi_2}}{2}, \quad (\text{A-7})$$

$$\begin{aligned} \varphi_1 &= \frac{(\lambda_c + 2\mu_s)(m\omega^2 - i\omega b)}{(\lambda_c + 2\mu_s)M - \alpha^2 M^2} + \\ &\quad \frac{\rho\omega^2 M - 2\rho_f\omega^2\alpha M}{(\lambda_c + 2\mu_s)M - \alpha^2 M^2}, \end{aligned} \quad (\text{A-8})$$

$$\varphi_2 = \frac{\rho\omega^2(m\omega^2 - i\omega b) - \rho_f^2\omega^4}{(\lambda_c + 2\mu_s)M - \alpha^2 M^2}, \quad (\text{A-9})$$

$$\zeta^2 = \gamma^2 + \lambda^2, \quad (\text{A-10})$$

$$\gamma^2 = \frac{\rho_f^2\omega^4 - (m\omega^2 - i\omega b)\rho\omega^2}{(m\omega^2 - i\omega b)\mu_s}, \quad (\text{A-11})$$

$$g_1 = -\frac{\rho_f\omega^2 + \alpha M \beta_1^2}{m\omega^2 - i\omega b + M \beta_1^2}, \quad (\text{A-12})$$

$$g_2 = -\frac{\rho_f\omega^2 + \alpha M \beta_2^2}{m\omega^2 - i\omega b + M \beta_2^2}, \quad (\text{A-13})$$

$$g_3 = -\frac{\rho_f\omega^2}{m\omega^2 - i\omega b}, \quad (\text{A-14})$$

$$C_{k1} = 2\lambda_n \xi_{1n} K_1(\xi_{1n} r_p) + 2\eta_{1n} \lambda_n \xi_{2n} K_1(\xi_{2n} r_p) - \eta_{2n} \zeta_n [(\lambda_n)^2 + (\zeta_n)^2] K_1(\zeta_n r_p), \quad (\text{A-15})$$

$$C_{k2} = \lambda_n K_0(\xi_{1n} r_p) + \eta_{1n} \lambda_n K_0(\xi_{2n} r_p) - \eta_{2n} (\zeta_n)^2 K_0(\zeta_n r_p), \quad (\text{A-16})$$

$$\eta_{1n} = \frac{\xi_{1n} K_1(\xi_{1n} r_p)(g_3 - g_1)}{\xi_{2n} K_1(\xi_{2n} r_p)(g_2 - g_3)}, \quad (\text{A-17})$$

$$\eta_{2n} = \frac{\xi_{1n} K_1(\xi_{1n} r_p)(g_2 - g_1)}{\zeta_n \lambda_n K_1(\zeta_n r_p)(g_2 - g_3)}, \quad (\text{A-18})$$

$$C_n = \frac{k_n}{\rho_p S_p [\omega^2 - (C_p \lambda_n)^2]}, \quad (\text{A-19})$$

$$F_{1n} = \int_0^{h_p} \sin\left(\frac{\omega}{C_p} z\right) \cdot \cos(\lambda_n z) dz, \quad (\text{A-20})$$

$$F_{2n} = \int_0^{h_p} \cos\left(\frac{\omega}{C_p} z\right) \cdot \cos(\lambda_n z) dz, \quad (\text{A-21})$$

$$F_{3n} = (1 - C_n) \int_0^{h_p} \cos^2(\lambda_n z) dz, \quad (\text{A-22})$$

$$\chi_n = \frac{-(\delta F_{1n} + F_{2n}) C_p}{\delta F_{3n} \omega E_p g_n}, \quad (\text{A-23})$$

$$\begin{aligned} g_n &= \lambda_n K_0(\xi_{1n} r_p) + \eta_{1n} \lambda_n K_0(\xi_{2n} r_p) - \\ &\quad \eta_{2n} (\zeta_n)^2 K_0(\zeta_n r_p). \end{aligned} \quad (\text{A-24})$$