#### DOI: 10.11779/CJGE201706025

# 关于"横观各向同性砂土的强度准则"的讨论

董彤1,孔亮2

(1. 后勤工程学院岩土力学与地质环境保护重庆市重点实验室,重庆 401311; 2. 青岛理工大学理学院,山东 青岛 266033)

## Discussion on "New strength criterion for sand with cross-anisotropy"

DONG Tong<sup>1</sup>, KONG Liang<sup>2</sup>

(1. Chongqing Key Laboratory of Geomechanics and Geoenvironmental Protection, Logistical Engineering University, Chongqing 401311,

China; 2. School of Sciences, Qingdao Technological University, Qingdao 266033, China)

中图分类号: TU431 文献标识码: A 文章编号: 1000 - 4548(2017)06 - 1161 - 02

作者简介:董 彤(1990 - ), 男, 山东新泰人, 博士研究生, 主要从事岩土本构关系方面的研究。E-mail: dt0706@126.com。

《岩土工程学报》2016 年第 38 卷 11 期刊出"横观各向同性砂土的强度准则"一文[1](以下简称"原文")。原文定义了一个新的无量纲各向异性参量  $\Lambda(\sigma,F)$ ,用于度量应力张量与组构张量的相对方位,利用该各向异性参量将 SMP 准则推广,得到一个新的适用于横观各向同性砂土的强度准则。拜读原文后,受益良多,同时认为存在可以进一步完善之处,在此指出,以期探讨。

基于岩土材料各向异性的物理机理,原文构造一个新的无量纲数  $\Lambda$  作为各向异性参量,其表达式为(参见原文式(12))

$$\Lambda(\boldsymbol{\sigma}, \boldsymbol{F}) = \frac{\sigma_{\text{N}} - \sigma_{\text{SMP}}}{I_{\text{L}}} \quad . \tag{1}$$

原文指出,由该各向异性参量所建立的各向异性本构模型的表达式是显式的,但  $\Lambda_{f0}$  与  $\Lambda_{f90}$  的求解方式原文并未给出;由于缺乏  $\Lambda$  、  $\Lambda^2$  与  $\delta$  之间函数关系的解析式,原文中图 5 与图 6 的绘制思路并不明确;假设  $\tan \Phi$  ' 与  $\Lambda^2$  具有线性关系的依据并不充分。

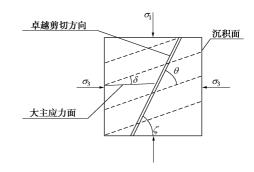
鉴于  $\sigma_{\rm N}$  (沉积面上正应力大小)、  $\sigma_{\rm SMP}$  (SMP 上正应力大小)以及  $I_{\rm I}$  (应力张量的第一不变量)所描述的是同一应力状态,三者之间必然存在一定的联系。

以二维情况为例,如图 1 所示,大主应力方向角  $\delta$  反映了沉积面与大主应力之间的位置关系;卓越剪切方向是剪切带的生成方向,具有唯一性,在二维情况下与 SMP 重合,与大主应力方向之间的夹角为  $\beta$  ,满足  $\beta$  =  $\beta$  。

因此,沉积面上应力 $\sigma_{
m N}$ 与 SMP 上 $\sigma_{
m SMP}$ 均可由主应力张量 $\sigma$ 通过坐标变换得到

$$\boldsymbol{\sigma}_{\mathrm{N}} = \boldsymbol{M}_{\mathrm{N}} \boldsymbol{\sigma} \boldsymbol{M}_{\mathrm{N}}^{\mathrm{T}} \quad , \tag{2}$$

$$\boldsymbol{\sigma}_{\text{SMP}} = \boldsymbol{M}_{\text{SMP}} \boldsymbol{\sigma} \boldsymbol{M}_{\text{SMP}}^{\text{T}} \quad , \tag{3}$$



#### 图 1 大主应力方向与卓越剪切方向示意图

Fig. 1 Angle of major principal stress and dominant shear direction 式中, $M_{\scriptscriptstyle \rm N}$ , $M_{\scriptscriptstyle \rm SMP}$  为转换矩阵,满足

$$\boldsymbol{M}_{\mathrm{N}} = \begin{bmatrix} \cos \delta & -\sin \delta \\ \sin \delta & \cos \delta \end{bmatrix}; \quad \boldsymbol{M}_{\mathrm{SMP}} = \begin{bmatrix} \cos \varsigma & -\sin \varsigma \\ \sin \varsigma & \cos \varsigma \end{bmatrix}. \quad (4)$$

将式(4)展开可得

$$\sigma_{N} = \sigma_{1} \cos^{2} \delta + \sigma_{3} \sin^{2} \delta , \qquad (5)$$

$$\tau_{N} = \sigma_{1} \sin \delta \cos \delta - \sigma_{3} \sin \delta \cos \delta , \qquad (6)$$

$$\int \sigma_{\text{SMP}} = \sigma_1 \cos^2 \zeta + \sigma_3 \sin^2 \zeta \quad , \tag{7}$$

$$\tau_{\text{SMP}} = \sigma_1 \sin \zeta \cos \zeta - \sigma_3 \sin \zeta \cos \zeta \quad . \tag{8}$$

二维情况下,应力不变量可以写作:

$$I_1 = (\sigma_1 + \sigma_3)/2$$
,  $\sqrt{3J_2} = \sigma_1 - \sigma_3$ , (9)

相应地, 主应力可以表示为

$$\sigma_1 = (2I_1 + \sqrt{3J_2})/2$$
,  $\sigma_3 = (2I_1 - \sqrt{3J_2})/2$  . (10)

因此,式(1)的解析形式为

$$\frac{\sigma_{\rm N} - \sigma_{\rm SMP}}{I_1} = \frac{\sqrt{3J_2}}{2I_1} (\cos 2\delta - \cos 2\varsigma) \quad , \tag{11}$$

将 $\varsigma = 45^{\circ} + \varphi_{cr}/2$ 代入式 (11), 得

$$\Lambda(\sigma, F) = \frac{\sqrt{3J_2}}{2I_1} (\cos 2\delta + \sin \varphi_{\rm cr}) \quad . \tag{12}$$

如原文图 5 所示, $\Lambda$ 与  $\delta$ 之间满足严格的三剪函数关系,当  $\Lambda$ = 0 时, $\delta$ = 45° + $\varphi_{\rm cr}/2$ ,即沉积面与 SMP 重合。与 Pietruszczak 等<sup>[2]</sup>、Kong 等<sup>[3]</sup>以及 Gao 等<sup>[4]</sup>相似,本文所定义的 各向异性参数也是为了描述大主应力方向与沉积面之间的位置关系,或主应力张量基矢与主组构张量基矢的位置关系。原文假设  $\tan \Phi$ '与  $\Lambda^2$  有单调关系,并进一步假设  $\tan \Phi$ '与  $\Lambda^2$  具有线性关系:

$$\tan^2 \Phi'(\Lambda) = \alpha + \beta \Lambda^2 \quad , \tag{13}$$

相当于假设强度参数与大主应力方向角之间满足特定的三角函数关系,根据已有试验规律该假设是合理的[1-4]。

总之,式(1)所定义的各向异性参量实为一个大主应力 方向角的三角函数,原文未将该解析式给出,望加以补充完善。

#### 参考文献:

[1] 曹 威, 王 睿, 张建民. 横观各向同性砂土的强度准则

[J]. 岩土工程学报, 2016, **38**(11): 2026 - 2032. (CAO Wei, WANG Rui, ZHANG Jian-min. New strength criterion for sand with cross-anisotropy[J]. Chinese Journal of Geotechnical Engineering, 2016, **38**(11): 2026 - 2032. (in Chinese))

- [2] PIETRUSZCZAK S, MROZ Z. Formulation of anisotropic failure criteria incorporating a microstructure tensor[J]. Computers and Geotechnics, 2000, **26**(2): 105 112.
- [3] KONG Y, ZHAO J, YAO Y. A failure criterion for cross-anisotropic soils considering microstructure[J]. Acta Geotechnica, 2013, 8(6): 665 673.
- [4] GAO Z, ZHAO J, YAO Y. A generalized anisotropic failure criterion for geomaterials[J]. International Journal of Solids and Structures, 2010, 47(22): 3166 - 3185.

### 第四届GeoShanghai国际会议通知

GeoShanghai is a series of international conferences on geotechnical engineering held in Shanghai every four years. The conference was inaugurated in 2006 and was successfully held in 2010 and 2014, with more than 1200 participants in total. Since the last conference, the geotechnical communities have witnessed many advances both in fundamental understandings and engineering practices. To show the latest developments and promote collaborations in geotechnical engineering and related topics, the organizers of the GeoShanghai International Conference would like to invite you to participate in the 4th GeoShanghai International Conference to be held in Shanghai in May 2018.

**Objective:** Provide a showcase of recent developments and advances and offer an international forum to discuss and explore the future directions for geotechnical engineering.

Themes: Soil behavior & geomechanics; Unsaturated soil mechanics; Seepage and porous mechanics; Rock mechanics and rock engineering; Pavement mechanics and engineering; Geohazards; Geosynthetics; Geoinformatics; Geotechnical in-situ testing & monitoring; Environmental geotechnics; Transportation geotechnics; Offshore geotechnics; Mining geotechnics; Energy-related geotechnics; Behavior of biotreated geomaterials and foundations; Geomechanics at macro & micro scales; Ground improvement; Soil dynamics & earthquake engineering; Landfills and contaminated soil; Sustainability in geotechnical engineering; Deep excavations & retaining structures; Shafts & deep foundations; Tunneling and underground constructions; Pavement materials

and structures; New frontiers in geotechnology; Case studies.

**Important Dates:** Abstract due: April 30, 2017; Acceptance of abstract: May 31, 2017; Full paper due: August 31, 2017; Acceptance of full paper: November 30, 2017; Final full paper due: January 31, 2018.

**Paper Submission:** Abstracts and full papers should be submitted only via the web in accordance with the instructions on the conference website at www.geo-shanghai.org.

**Proceeding:** All the accepted papers will be included in a published proceeding to be submitted for citation by EI. Excellent papers will be published in special issues of several international journals to be submitted for citation by SCI based on peer-review.

**Exhibition:** Lab, field testing, instrumentation, geosynthetics, and other geotechnical manufacturers, suppliers, contractors, installers, and consulting firms are welcome to exhibit.

**Conference Website:** Please visit the website at www.geo-shanghai.org for updates and more detailed information.

Contacts: Ming Xiao, Ph.D., P.E., Associate Professor Department of Civil & Environmental Engineering, The Pennsylvania State University, University Park, PA 16802, USA Tel: 814-867-0044, Email: mxiao@engr.psu.edu. Xiaoqiang Gu, Ph.D., Assistant Professor Department of Geotechnical Engineering, College of Civil Engineering, Tongji University 1239 Siping Road, Shanghai 200092, China Tel: + 86(21)-6598-4551, Fax: + 86(21)-6598-5210 E-mail: geoshanghai@tongji.edu.cn.